using implicit method of characteristic lines (IMOC) has shown that it will be helpful to use implicit method of characteristic lines instead of the explicit characteristic lines method in order to lower and balance the limitation [8]. In another research, the effect of size in pressurized air reservoir in reducing maximum and minimum pressure due to water hammer has been studied. The research on optimization of conveying systems with pumps for water hammer using mathematical optimization method had shown that within increasing pipe diameter, effect of sudden pump stoppage especially negative pressure will be lowered. Within this method, the diameter and thickness of pipe will be optimized in order to prevent water hammer occurrence and unnecessarily expenses [9]. The hydraulic simulation study on water hammer using multiple diameters and materials of pipes showed that changes in material must be in order of the pipe with higher elasticity module to the pipe with lower elasticity module. That is, selection of closer elasticity module for pipe segments, resultsin lower pressure changes [10]. Comparison for control of transient hydraulic waves of water hammer showed that protective actions and design of expansion joints is based on low flow velocity, using check valves, control valves, balancing reservoirs and air reservoirs. In another research about water hammer in hydroelectric power plants, numerical analysis of water hammer had significant impact on the output of the actual projects. However, tolerances are visible due to simplifications and inaccessibility of some required data. Assessment of water hammer simulation using laboratory and numerical CFD models showed that numerical CFD simulation model of water hammer has high reliability and can be used as a proper numerical model to calculate maximum and minimum pressure. Mutual assessment between water hammer and centrifugal pumps showed that the centrifugal pumps especially in high energy level and velocity generate remarkable pressure fluctuations. Interaction effect can increase the effects, so that the pressure fluctuation should not be neglected [11]. Assessment of water hammer simulation using implicit method of characteristics represents high reliability of the method, which can simulate discharge and water levels in all considered cases [12].

In another study, critical hydraulic gradient for sediment transport through rockfilldam was determined [13]. Results from dynamic pressure fluctuations in stepped three-side spillway showed that the proposed form of ogee profile caused a significant reduction in turbulence intensity within the side channel. On the other hand, the stepped Ogee profiles of three-side spillways caused to simple construction and ease of operation [14].In a research, the effect of temperature and influent loadon nitrifying treatment of wastewater using CFD has been conducted [15].

In this study, the purpose is to solve the governing equations about water hammer phenomenon and analysis of the sensitivity of some hydraulic parameters. For this purpose, a program in MATLAB Environment was prepared. Fluctuations of pressure by changing of pipe diameter, length and velocity, were investigated. Sensitivity analysis of the numerical model by changing parameters, contributes to well understanding about water hammer [16, 17].

## MATERIALS AND METHODS

**Governing Equation:** The general equation of water hammer is obtained from Newton's second law and the equation of continuity of flow. Eq. 1 is known as Euler equation or the momentum equation. This equation is used for non-compressible fluids.

$$\frac{1}{\rho} \cdot \frac{\partial P}{\partial x} + \frac{fV|V|}{2D} + g \cdot \sin \alpha + V \frac{\partial V}{\partial x} + \frac{\partial V}{\partial t} = 0$$
(1)

In Eq. 1, parameter D is internal diameter of pipe, P is pressure, x is location dimension, t is time dimension, f is friction coefficient, V is average flow velocity and L is pipe length.

Applying continuity equation considered for an element of pipe length, results in Eq. 2.

$$a^{2} \cdot \frac{\partial V}{\partial x} + \frac{1}{\rho} \frac{\partial P}{\partial t} + \frac{1}{\rho} V \frac{\partial P}{\partial x} = 0$$
(2)

The Eq. 2 would be used simultaneously with Eq. 1 to solve water hammer phenomenon. In addition, a in Eq. 2 is velocity of pressure waves.

The Characteristic Lines Method for Numerical Solution: History of water hammer analysis is an implication for various methods development to solve Euler and continuity equation (Eqs. 1 and 2). The variety of these methods is depended on numerical analysis ability and innovation of these methods. The characteristic lines method is one of the most accurate methods to assess water hammer phenomenon because it considers minor losses and also it is customizable for various boundary conditions. In this method, the partial differential equations of flow continuity and momentum



Fig. 1: Characteristic lines



Fig. 2: The system consists of a simple pipe with a reservoir at upstream and a valve in downstream of the pipe

convert to the two ordinary differential equations and then could be solved by finite difference method [5]. By performing some mathematical operations, two ordinary differential equations are obtained as Eqs. 3 and 4.

$$\frac{\partial H}{\partial t} + \frac{c}{gA}\frac{\partial Q}{\partial t} + \frac{cf}{2gD}V|V| - \frac{Q}{A}\sin\alpha = 0, \frac{\partial x}{\partial t} = 1 + c$$
(3)

$$\frac{\partial H}{\partial t} - \frac{c}{gA}\frac{\partial Q}{\partial t} + \frac{cf}{2gD}V|V| - \frac{Q}{A}\sin\alpha = 0, \frac{\partial x}{\partial t} = 1 - c$$
(4)

Eqs. 3 and 4 are established on lines  $\frac{\partial x}{\partial t} = 1 + c$  and  $\frac{\partial x}{\partial t} = 1 - c$ . Eqs. 3 and 4 on coordination screen of (x-t) are explainer of two straight lines of 1/c, -1/c. Thus, the differential equation on these lines using finite difference method can be written as follows (Fig. 1):

B,  $C_m$ ,  $C_p$  are known as coefficients based on value of H and Q in time step (n is present time). By solving these two linear equations, the two unknown values for  $Q_i^{n+1}$ ,  $H_i^{n+1}$  in the next time step will be found.

In this study, a computer program in MATLAB environment was presented to solve the governing equations of water hammer (momentum and continuity of flow). The prepared program solves transient fluctuations in a simple pipeline, with an upstream reservoir and a downstream valve (Fig. 2). The valve specification places as  $C_DA$  in orifice formula (Eq.5).

$$Q_P = C_D A \sqrt{2gH_P} \tag{5}$$

Specifications of the system that MATLAB program was designed stated as follows:

where,  $H_p$  is reservoir water levels, L is pipe length, f is pipe's friction coefficient and a is the velocity of wave.

The datum for hydraulic levels is considered to be the geometrical axis of the pipe. The program in each time steps calculates the value of  $C_DA$  which is CV in program using linear interpolation. Simultaneously, the value of  $H_p$  and  $Q_p$  in valve would be calculated by solving Eq. 5 and characteristic equation of  $C^+$  (Eq. 3). To specify the permanent conditions for energy equation from reservoir to valve, neglecting minor losses will expressed as follows:

$$H_R - f \frac{L}{D} \frac{Q_0^2}{2gA^2} = \frac{Q_0^2}{2gCV^2}$$
(6)

## **RESULTS AND DISCUSSIONS**

In this section the behavior of water hammer on a system including a pipe within variable diameter and length and wave's variable velocity within constant 100 meters head of reservoir, would be assessed. For this purpose, a code in MATLAB language has been written which in the parameters are allowed to be replaced and plotted. Method to solve the governing equations is the characteristics method. The fluctuation of pressure is calculated in 4 statuses (pipe's full length, pipe's  $\frac{3}{4}$  length, pipe's  $\frac{2}{4}$  length and pipe's  $\frac{1}{4}$  length). For brevity the results are mentioned for diameters of 2 and 3 meters in the Figs. 3 and 4.

According to Figs. 3 and 4, it is clear that when the diameter is considered as variable, within diameter increment, the pressure fluctuation range decreased. The reason is that in bigger diameters the cross-sectional area of pipe is bigger, so that the pressure differences would distribute on this (bigger) area.

Fig. 3 shows that the maximum pressure increment in pipe with diameter of 2 metersis about 62% of the static head of reservoir and for pressure decrement it is about





Fig. 3: Pressure fluctuations in different positions of pipe with diameter of 2 meters



Fig. 4: Pressure fluctuations in different positions of pipe with diameter of 3 meters



Fig. 5: Pressure fluctuations at the end of pipe for diameters of 0.1 to 10 meters

33.35% of the static head of reservoir. Thus, controlling pipe diameters in order not to break pipes and also controlling the danger of cavitation due to pressure decrement should be considered by designer [13]. In addition, according to Figs. 3 and 4, it is clear that the maximum and minimum pressure occur at the end of pipe, so that the end of pipe is considered as critical zone in design criteria.

The pressure fluctuations for middle and end of the pipes within diameters of 0.1 meter to 10 meters are presented in Figs. 5 and 6.

According to Figs. 5 and 6 it could be extracted that within diameter increment the range of pressure fluctuation decreased and as a result the energy dissipation occurred faster. In the other words, with diameter increment the transiency of flow would dissipate promptly. It is clear that the designer must consider the expenses of the bigger diameter and must prepare the optimized design for decreasing pressure and decreasing expenses of purchasing and setting up the pipeline.

In the next phase, the length is variable and other parameters are constant. For brevity the results for length of 3800 meters and 5800 meters are presented in Figs. 7 and 8.

According to Figs. 7 and 8, with length increment the range of pressure fluctuations would increase. Thus, designer must choose the shortest distance to lower expenses of pressure waves control and pipe's own expenses.





Fig. 6: Pressure fluctuations in the middle of pipe for diameters of 0.1 to 10 meters



Fig. 7: The pressure fluctuations for pipe within length of 3800 meters



Fig. 8: The pressure fluctuations for pipe within length of 5800 meters

Accurate assessment for length effect on water hammer is presented in Figs. 9 and 10.

It is observable from Figs. 9 and 10 that with length decrement, the pressure fluctuation range decreases. The reason is that, in the shorter pipes the pressure waves sweep more and rapid and this cause to encounter the pressure waves in opposite direction, therefore it causes more dissipation of waves.

Now if the velocity of wave considered as variable, Figs. 11 and 12 are resulted.

According to Figs. 11 and 12 with velocity increment from 1000 to 1400 (m/s), the pressure fluctuation range decreases. The reason is that in higher wave velocity the wave's sweep occurs more and rapid and this could encounter waves in opposite direction.

Figs. 13 and 14 present the effect of different wave's velocity for middle and the end of the pipe.

Again within an exact look at Figs. 13 and 14, it can be seen that increasing wave velocity would results in more increasing transiency. The end of the pipe is critical



Fig. 9: Pressure fluctuations at the end of the pipe for different lengths of 100 to 5800 meters



Fig. 10: Pressure fluctuations in the middle of the pipe for different lengths of 100 to 5800 meters



Fig. 11: Pressure fluctuation in pipe with wave's velocity of 1000 (m/s)



Fig. 12: Pressure fluctuation in pipe with wave's velocity of 1400 (m/s)





Fig. 13: Effect of different wave's velocity on positive and negative pressure at the end of pipe



Fig. 14:Effect of different wave's velocity on positive and negative pressure in the middle of pipe

zone for water hammer phenomenon, because absolute max/min pressure in end of pipe is more. Finally, design engineer must consider it as a critical point of project.

## CONCLUSION

- With increasing pipe diameter, the pressure fluctuation range would be small. In the other words the transiency of waves would be more.
- The pressure fluctuation range would remarkably decrease by using shorter pipes.
- With wave's velocity increment, the pressure fluctuation range would decrease.
- The maximum and minimum pressure occurs at the end of the pipe. Thus, end of the pipe is critical point in design criteria.

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