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Form-Finding of Tensioned Fabric Structure in the Shape of Möbius Strip

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Abstract: Form-finding of tensioned fabric surface bordered by Möbius strip is investigated. Möbius strip is a surface with only one side and only one boundary component. Möbius strip has the mathematical property of being non-orientable. The form of Möbius strip has been adopted in the creation of sculpture, exploration of idea for bridges and buildings. In this study, the possibility of adopting the form of Möbius strip as surface shape for tensioned fabric structure has been studied. The combination of shape and internal forces for the purpose of stiffness and strength is an important feature of tensioned fabric surface. For this purpose, form-finding needs to be carried out. In this study, nonlinear analysis method is used for form-finding analysis. The influence of non-orientable characteristic of Möbius strip on the modeling process using finite element procedures was investigated. Pattern of pre-stress in the resulting tensioned fabric surface is also studied. Form-finding has been found to converge for Möbius strip R/W $\leq$1.37 with initial assumed shape specified to follow the topology without opening which has been observed in experiment.

Key words: Form-finding · Tensioned fabric structure · Möbius strip · Non-orientable · Nonlinear analysis method

INTRODUCTION

Tensioned fabric structures are normally designed to be in the form of equal tensioned surface. Classical minimal surface such as Catenoid, Helicoid, Enneper and Scherk or their variation have been much studied as possible choice of surface form for tensioned fabric structure (TFS). TFS is highly suited to be used for realizing surfaces of complex or new forms. One of the most interesting surface in the world of mathematics is the so-called Möbius strip. It is a one-sided non-orientable surface. However, none of the examples mentioned present any results on the form of Möbius strip as load carrying members. One example closest to the idea of using the form of Möbius strip as load carrying members is the children playground. However, such climber structure in children playground lacks the feature of being a continuous strip which is one of the principal reasons why Möbius strip has generated so much interest.

Despite the interesting feature of one-sided surface associated with Möbius strip, research study on its behaviour as a structure has not attracted much attention. Mahadevan and Keller [1] have represented the Möbius strip as a bent, twisted elastic rod with a rectangular cross-section. These are solved numerically for various values of the aspect ratio of the cross-section and an asymptotic solution is found for large values of such ratio. Kendall and Michael [2] have theorized through the metaphor of the Möbius strip. Petresin and Robert [3] have shown that the Möbius strip has a great potential as an architectural form. Other researchers [4] have investigated the influence of the topology on generic features of the persistent current in n-fold twisted Möbius strips formed of quasi one-dimensional mesoscopic rings; both for free electrons and in the weakly disordered regime. Tanda *et al*. [5] have fabricated a microscopic NbSe3 Möbius strip and this raises some interesting questions regarding the topological effect on quantum

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transport. The topological effect on quantum transport for isolated Möbius strips has been described in the literatures [6, 7]. Maiti [7] has studied electron transport through a Möbius strip attached to two metallic electrodes by the use of a Green's function technique. Choong and Kuan [8] have carried out a study on the possibility of adopting the idea of Möbius strip as a shell structure. Structural behaviour under different support conditions was investigated. The study concluded that support conditions played an important role on the strength and stiffness of shell in the form of Möbius strip.

The form of Möbius strip has been adopted in the creation of sculpture [9]; also the exploration of idea for bridges and for buildings discuused by Séquin and Carlo [10]. Lewekea *et al*. [11] have mentioned that Möbius strip presents a profile that is locally a flat plate with an angle of attack smoothly varying between -90° and 90° and this regardless of the orientation of the object. They [11] have represented results from an experimental study of Möbius bands in free fall, focusing on the trajectory, body motion and wake dynamics. Extensive work related to the subject reported in the literatures [12-16]. Idea of Mobius strip as structures has also been mentioned in other studies [17]. However, detailed investigation on Mobius strip as a structure has not been pursued. TFS in the form of Möbius strip have not been studied by other researchers. This study has been carried out as an initial study on characteristics of initial equilibrium shape of tensioned fabric structures in the form of Möbius strip minimal surface. Understanding of the possible initial equilibrium shapes to be obtained will provide alternative shapes for designers to consider. Before the shapes can be considered for structural application, their behaviour under load must be properly studied. For the case of tensioned fabric structures, the first step in any structural analysis is the determination of initial equilibrium shape. Factors affecting initial equilibrium shape in the form of Möbius strip minimal surface need to be studied.

Generation of Möbius Strip Tensioned Fabric Surface: Listing [18] and Möbius [19] have stated that Möbius strip was ''invented'' independently in 1858 by them (German mathematicians). Mahadevan and Keller [1]; Starostin and Heijden [20] have mentioned that a Möbius strip made from a rectangular strip will assume a complicated shape in space, depending on the aspect ratio and thickness of the initial rectangle and the elastic properties of the material. Horowitz and Williamson [16] have mentioned that Möbius strip have already been observed to be a central feature. A Möbius strip of half-width W with midcircle of radius R at height z = 0 as shown in Fig. 1 is represented parametrically by the following set of equation by Weisstein and Eric [21]:

$$X = (R + S\cos\frac{\theta}{2})\cos\theta, Y = (R + S\cos\frac{\theta}{2})\sin\theta$$

$$Z = S\sin(\frac{\theta}{2}) \qquad (1)$$

for {S:-W,W}and {θ:0,2π}.

The problem with generation of model in the form of Möbius strip TFS model is that it is a single continuous surface involving only one curve along the band edges in a Cartesian space. The surface of a Möbius strip created by Eq. (1) stated in the literature [21] has the normal of the faces at the adjoining edges of the band in opposite direction as shown in Fig. 2 which makes it impossible to join the band borders thus closing the surface. Schwarz [22] also has mentioned that the surface defined by Eq. (1) has the disadvantage of not being developable. This means that, in principle, one cannot find a two-dimensional flat shape that could be twisted and glued together to give precisely this Möbius strip have been mentioned by Lewekea [11]. It was therefore decided to settle for a sufficiently close approximation of this shape, the solution adopted was to introduce a small gap (1°) across the width of the strip as depicted in Fig. 2. In this way, the surface becomes discontinuous over the gap.

Numerical Analysis: Different combination of parameters R and W for models of Möbius strip TFS models have been studied. The determination of R/W has been carried out as follows: i. R is fixed and W is increased and ii. W is fixed and R is increased. Möbius strip TFS models with R/W ≤ 0.4-10 have been studied with 0.1 increment for R and W. Between the range of R/W 1.2 to 1.4, R and W with 0.01 increment has been used. When R/W ratio is increased, the opening of Möbius strip model becomes larger. Form-finding has been found to converge when R/W > 1.37. For Möbius strip model with R/W ≤ 1.37, divergence is observed in form-finding and the shape obtained after form-finding are found to be different from the form of Möbius strip surface. Variation of root mean square deviation (RMSD) for Möbius strip TFS model has been found to converge.

Case a (R/W = 1) case b (R/W = 1.5) and case c (R/W = 2) have been chosen for discussion from models with R/W = 0.4-10 of Möbius strip TFS models. The surface of a Möbius strip generated by Eq. (1) for Möbius

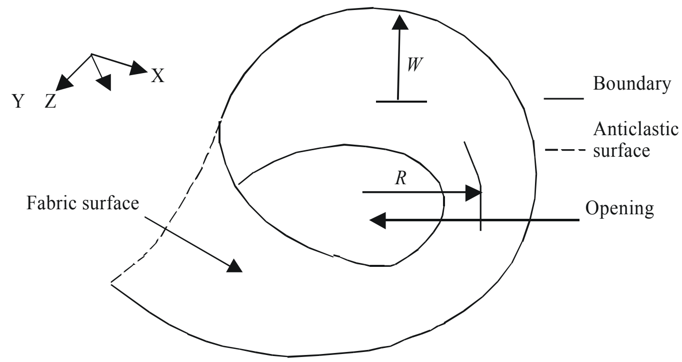


Fig. 1: Möbius strip TFS model

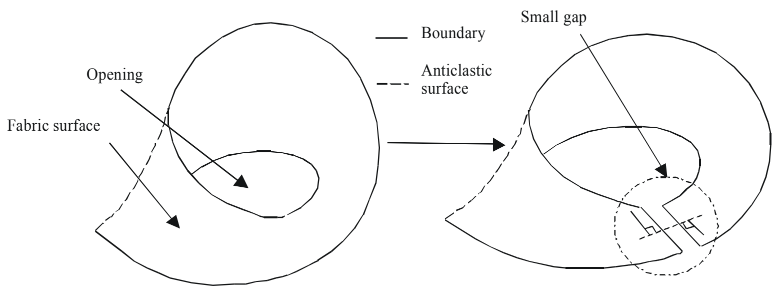


Fig. 2: Small gap of fabric surface in the form of Möbius strip TFS model

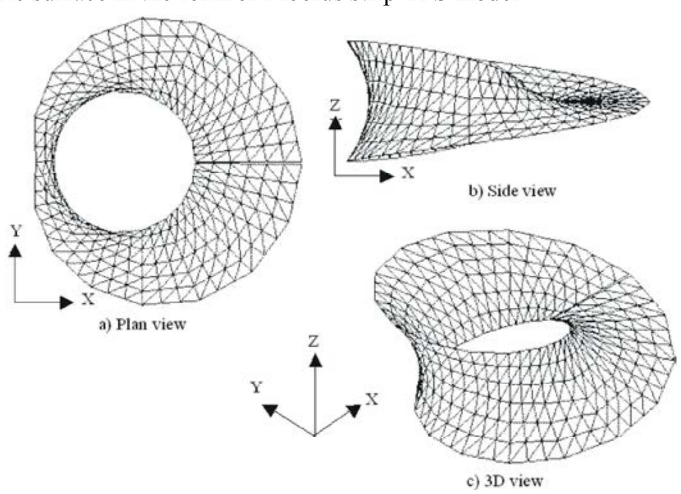


Fig. 3: Möbius strip TFS model after form-finding (case c (R/W = 2)).

strip TFS models case a (R/W = 1), case b (R/W = 1.5) and case c (R/W = 2) have a total number of 343 nodes and 576 elements. The member pretension in warp and fill direction, denoted as σW and σF respectively, is 2000N/m. Fig. 3 shows the converged shape of the Möbius strip TFS models after form-finding. Similar converged shape as that in case c (R/W = 2) shown in Fig. 3 has also been observed in case b (R/W = 1.5). The surface model of case a (R/W = 1) has been found to intersect with each other after form-finding as shown in Fig. 4a. Fig. 4b shows a closer view of intersection of surface. The corresponding variation of least square error (LSE) versus analysis stage as shown in Fig. 4c clearly indicates that the form-finding analysis has diverged. Such significantly highly LSE shows that the form-finding analysis does not converge. All elements have warp to fill stress ratio of approximately equal to one. The ratio of warp to fill stresses, σW/σF obtained for the models of case b (R/W = 1.5) and case c (R/W = 2) are approximately equal to one. This shows that the obtained shapes are equal tensioned surface or minimal surface. This results also show that the proposed computational strategy is able to yield accurate initial equilibrium shape for Möbius strip TFS models with case b (R/W=1.5) and case c (R/W=2).

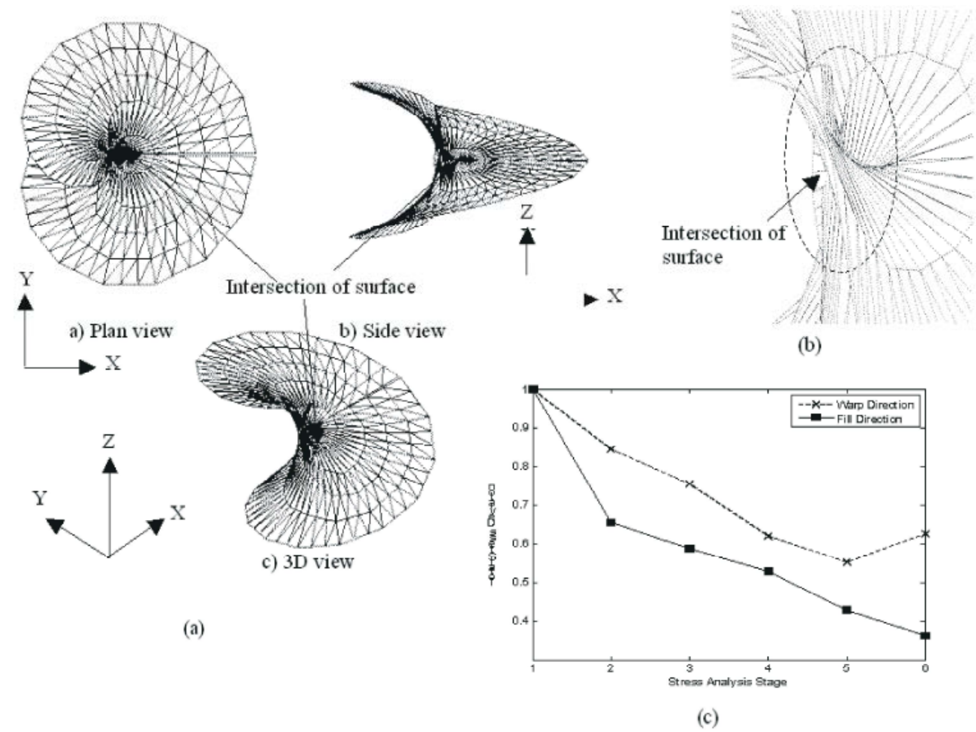
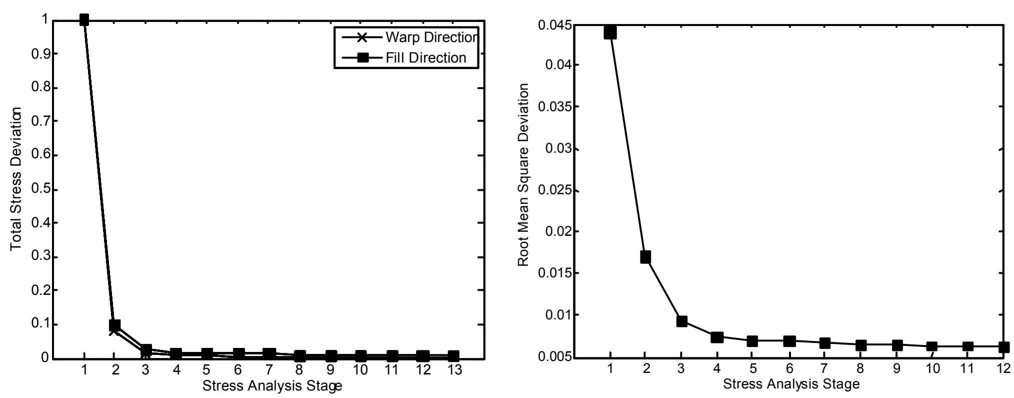


Fig. 4: Möbius strip TFS model after form-finding (case c (R/W = 2)). a) Models of membrane surface intersection with each other in the form of Möbius strip after form-finding (case a (R/W = 1)) (b) Intersection of surface (c) Variation of total stress deviation in warp and fill direction versus stress analysis stage for the Möbius strip TFS model (case a (R/W = 1))



(a) (b)

Fig. 5: (a)Variation of total stress deviation in warp and fill direction versus stress analysis stage for Möbius strip TFS model (b) Variation of RMSD versus stress analysis stage for the Möbius strip TFS model (case a (R/W = 2))

Variation of total stress deviation in warp and fill direction versus stress analysis stage for Möbius strip TFS model case c (R/W = 2) is shown in Fig. 5a. The same variation of total stress deviation in warp and fill direction versus stress analysis stage in case c (R/W = 2) has also been observed in case b (R/W = 1.5). Fig. 5b shows variation of RMSD versus stress analysis stage for the Möbius strip TFS model case c (R/W = 2). The convergence towards the mathematical shape can be clearly seen in the Fig. 5b.

When R/W $\leq$ 1.37, divergence has been observed in form-finding. The shape obtained in case a (R/W = 1) after form-finding is found to be different from the form of Möbius strip surface with no opening at the center as

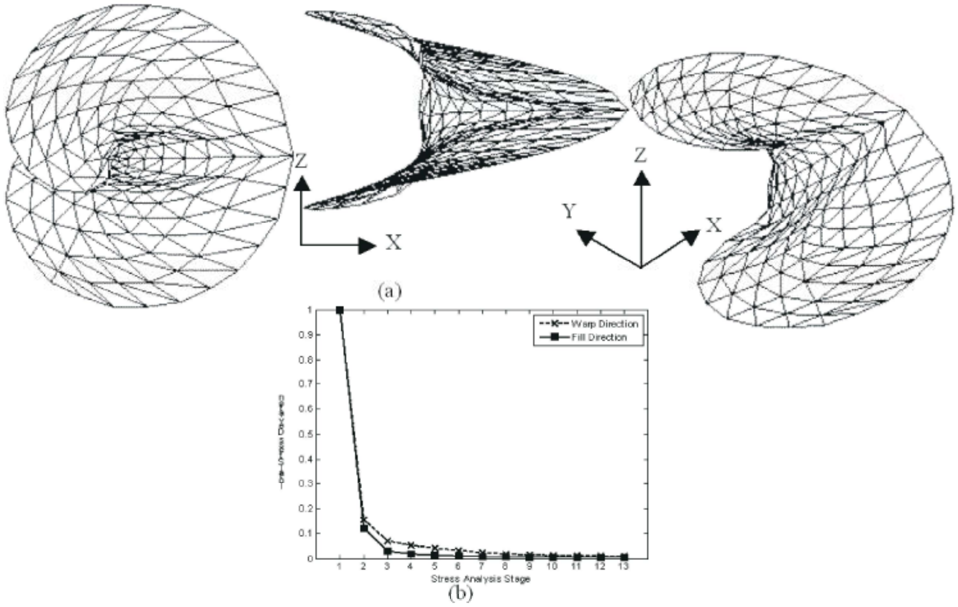


Fig. 6: (a) Möbius strip (without opening) TFS model after form-finding (case e (R/W = 1)) (b) Variation of total stress deviation in warp and fill direction versus stress analysis stage for the Möbius strip (without opening) TFS model (case e (R/W = 1))

Fig. 7: (a) Soap film model (b) Comparison of experimental and computational results for Möbius strip case e (R/W = 1).

illustrated in Fig. 4a. It can be seen that initial equilibrium shape of Möbius strip model (with opening at the center) for R/W = 1.37, convergence cannot be achieved due to intersection of surface Figs. 4a and 4b.

Another set of analysis of the Möbius strip models case d (R/W = 0.7143), case e (R/W = 1) and case f (R/W = 1.37) have been carried out by assuming the initial shape during model generation to be in the form similar to that observed in Fig. 4a. The Möbius strip (without opening) models of case d (R/W = 0.714), case e (R/W = 1) and case f (R/W = 1.37) have total number of 171 nodes and 300 elements. Similar converged shape of case e (R/W = 1) has been observed in case d (R/W = 0.714) and case f (R/W = 1.37). The ratio of warp to fill stresses, $\sigma W/\sigma F$ obtained for the Möbius strip (without opening) models of case d (R/W = 0.714), case e (R/W = 1) and case f (R/W = 1.37) is 1.0028, 0.9986 and 0.999, respectively. Fig. 6a also shows the convergent curve of the Möbius strip (without opening) model with case e (R/W = 1). The convergent curve shows that the total warp and fill stress deviation <0.01.

The characteristics of Möbius strip with opening at the center as shown in Fig. 3 has not been observed in the soap film model of Möbius strip case e (R/W = 1). It has been shown that form-finding by assuming an initial shape with the opening at the center is not able to yield converged result as shown in Fig. 4a. Hence, comparison with mathematically defined surface of Möbius strip case e (R/W = 1) is not possible due to the reason the mathematically defined surface has opening in the center. For the purpose of confirmation, the initial assumed shape for computational analysis has been generated in such a way that the topology of no opening as observed in experiment is followed. The computational result has been found to be in similar shape to that observed in soap film model. Fig. 7a shows the soap film model of Möbius strip case e (R/W = 1). The shapes of soap film model and computational results are found to be in good agreement in terms of their shapes as illustrated in Fig. 7b.

RESULTS AND DISCUSSIONS

A series of Möbius strip TFS models with ratios of R/W ranging from 0.4 to 10 were analysed. From the results of form-finding, it has been found that for Möbius strip with R/W >1.37, the form obtained is similar to the Möbius strip surface with opening at the center of the surface. The LSE of total warp and fill stress deviation satisfies the tolerance of 0.01. Values of RMSD obtained are found to be not more than 0.3 which indicates that the deviation is negligible considering the size of Möbius strip models used.

When R/W is = 1.37, divergence occurred. However, it was observed in form-finding. The shape obtained after form-finding are found to be different from the form of Möbius strip surface as shown in Fig. 4 where the opening at the center is missing. For cases of Möbius strip TFS models with R/W = 1.37, intersection of surface has been observed as shown in Fig. 4b. Caurant [23] has stated that Möbius strip is not able to keep the topological character of an opening at certain values of parameter. The results of R/W = 1.37 show that convergence of form-finding under the assumption of initial assumed shape with opening at the center is not able to be achieved. Form-finding has been found to converge for Möbius strip TFS with R/W = 1.37 by assuming the initial assumed shape to follow topology without opening. Fig. 6a shows the converged shape of Möbius strip TFS model without opening.

CONCLUSION

Form-finding of Möbius strip TFS models by assuming an initial assumed shape with the opening at the center is not able to yield converged result for the value of R/W = 1.37. The shape obtained after form-finding are found to be different from the form of typical Möbius strip surface with opening at the center. The opening at the center of Möbius strip is found to be non-existent. Shape of Möbius strip (without opening) has been verified through soap film model with R/W = 1. Form-finding has been found to converge for Möbius strip TFS model (with R/W = 1.37) with initial assumed shape specified to follow the topology without opening which has been observed in experiment.

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