

Nonlinear Thermal Analysis of Solar Air Heater for the Purpose of Energy Saving

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Abstract: In this article, nonlinear thermal Analysis of air-heating flat-plate solar collectors was conducted to obtain temperature distribution inside the heat transfer unit. Due to temperature-dependency of specific heat coefficient of air, the governing differential equation had a noticeable nonlinear nature. Therefore, for the solution of nonlinear equation a highly accurate and simple analytical method which is called Differential Transformation Method (DTM) and basic numerical method, Runge-Kutta (RG) was employed. As a significant result, it is depicted that the DTM results have an interesting conformity with numerical ones. After this verification, the effects of physical applicable parameters on temperature distribution, useful gained heat and thermal efficiency of collectors were investigated. It was concluded that increase in dimension of collector may not increase the efficiency (about 10% decreased). However an increase in the effectiveness coefficient ($\tau\alpha$) and air mass flow rate (\dot{m}) enhanced the performance of solar collectors and the process efficiency increased up to 12%.

Key words: Air-heater • Differential Transformation Method • Temperature Dependent Parameter • Thermal Efficiency • Solar Collector

INTRODUCTION

In recent years, special attention is paid to renewable energy resources which are alternative replacement of fossil fuels. One of the clean renewable energy is solar energy which is used for heating and cooling in buildings, drying process, water heating in domestic and industrial applications, heating swimming pools, generate electricity, many chemical applications and other operations [1].

One of the useful applications of solar energy is air heating which is shown in Fig. 1. Solar air heating (SAH) is a solar thermal technology in which the energy from the sun, solar insulation, is captured by an absorbing medium and used to heat air. SAH as a renewable energy, the technology is used for air conditioning and often used for heating purposes. It is typically the most cost-effective solar technologies, especially in commercial and industrial applications and it addresses the largest usage of building energy in heating climates, which is space heating and industrial process heating. The performance of solar air heaters is mainly influenced by the several parameters such as: meteorological parameters (direct and diffuse

radiation, ambient temperature and wind speed), design parameters type of collector, collector materials and flow parameters (air flow rate, mode of flow). The principal requirements of these designs are a large contact area between the absorbing surface and air [1].

The solar air heating collectors can be used into two main categories for drying and heating the room. Product issues through drying in sun and hot air are discussed in literatures [2]. The SAH system consists of two parts: a solar collector mounted on the side of the building facing the equator and a fan and air distribution system installed inside the building (Fig. 1). SAH collectors can be classified into two types, named bare-plate and covered plate solar energy collectors.

Bare-plate collectors are the simplest forms of solar collectors which they consist simply of an air duct, absorber plate and an insulated surface [3]. Covered-plate collectors are used for minimizing the upward heat losses. Common cover materials used are glass, Plexiglas and clear plastics. The cover material prevents convective heat losses from the absorbing plate, reduces radiative heat losses and protects the absorber plate against

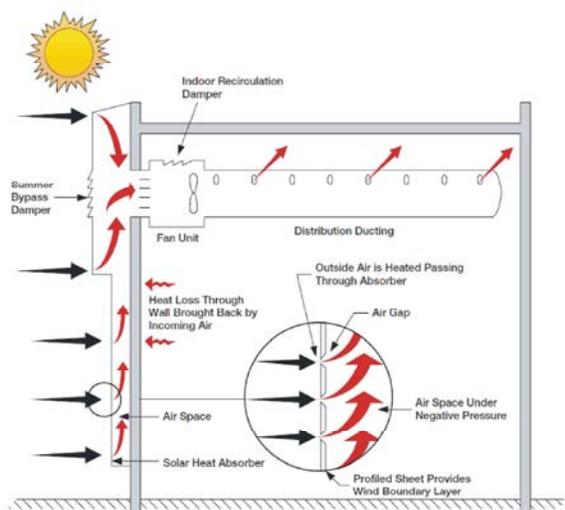


Fig. 1: Schematic diagram of the solar collector application

cooling by irregular rainfall [4]. However, the cost of cover materials and construction for covered-plate collectors is increasing but they operate at higher efficiencies than bare-plate collectors at moderate temperatures [5].

There are various design configurations available involving different combinations of geometrical details and airflow patterns. A novel solar air collector of pin-fin integrated absorber is designed to increase the thermal efficiency by Peng *et al.* [6]. Their design has many advantages such as achieving high thermal efficiencies, large flexibility, having long durability and acceptable costs. Review of various designs and performance evaluation technique of flat-plate SAH collectors for low temperature solar-energy crop drying applications and appropriateness of each design and the selection of materials as guidelines have been prepared by Ekechukwu [4].

Dovic and Andrassy [7] presented the 2-D and 3-D numerical finite volume models for collectors which allow the precise analysis of important design and operating parameters such as dimensions and shape of the tube bond, coating and emissivity, glazing to absorber distance, tube diameter, fluid flow rate and wind velocity. Their simulations performed on collectors without tubes and the results showed that no significant improvement of its efficiency can be achieved by either decreasing or increasing the distance between absorber and glazing. The numerical method of solving the set of first-order partial differential model equations which presented by Hilmer and Vajen is applicable for collector performance assessment at the time of varying fluid flow rate, solar

irradiance and heat transfer coefficients [8]. Leon and Kumar [9] developed a mathematical model for predicting the thermal performance of unglazed transpired collectors (UTC) over a range of design and operating conditions suitable for drying of food products.

There are a lot of methods to solve the nonlinear equations of engineering cases [10, 11]. Differential transform method (DTM) is a useful and applicable method to overcome the complexity of such problems. DTM is based on Taylor series expansion. It constructs an analytical solution in the form of a polynomial. It is different from the traditional high order Taylor series method, which requires symbolic computation of the necessary derivatives of the data functions. This method was first applied in engineering domain by Zhou [12] and its abilities have attracted many authors to use this method for solving nonlinear equations. Borhanifar *et al.* [13] employed DTM on some PDEs and their coupled versions. Abdel-Halim Hassan [14] has applied the DTM for different systems of differential equations and he has discussed the convergence of this method in several examples of linear and non-linear systems of differential equations. Abazari and Abazari [15] have applied DTM for solving the generalized Hirota-Satsuma coupled KdV equation. Abbasov *et al.* [16] employed DTM to obtain approximate solutions of the linear and non-linear equations related to engineering problems and they showed that the numerical results are in good agreement with analytical solutions. Balkaya *et al.* [17] applied DTM to analyze the vibration of an elastic beam supported on elastic soil.

In this study, the authors' intention was to obtain and solve the nonlinear differential equation of the air-heating flat-plate solar collector to find temperature distribution inside the unit. For this aim, analytical differential transformation method (DTM) and numerical method were applied and the effects of effective parameters (W , L , U_L , ta and m^*) appearing in the formulation on the collector thermal efficiencies were investigated.

MATERIALS AND METHODS

Problem Description: Fig. 2 shows a control volume of a solar collector which contains absorber plate, air medium and back plate. An energy balance on the absorber plate of area ($1 \times \delta x$) is given by equation [1]:

$$S(\delta x) = U_t(\delta x)(T_p - T_a) + h_{c,p-a}(\delta x)(T_p - T) + h_{r,p-b}(\delta x)(T_p - T_b) \quad (1)$$

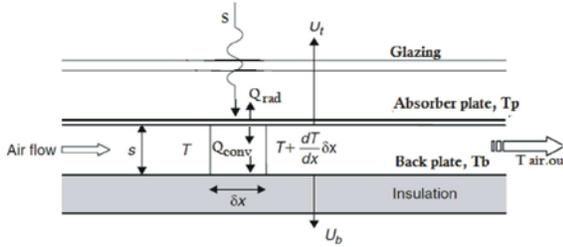


Fig. 2: Control volume for energy balance of absorber plate, back plate and air flow

Where $S=(\tau\alpha).G_c$. An energy balance of the air stream volume ($s \times 1 \times \delta x$) is given by the following equation:

$$\frac{\dot{m}}{W} c_p \left(\frac{dT}{dx} \delta x \right) = h_{c,p-a} (\delta x) (T_p - T) + h_{c,b-a} (\delta x) (T_b - T) \quad (2)$$

An energy balance on the back plate area ($1 \times \delta x$) is given by the following equation;

$$h_{r,p-b} (\delta x) (T_p - T_a) = h_{c,b-a} (\delta x) (T_b - T) + U_b (\delta x) (T_b - T_a) \quad (3)$$

As U_b is much smaller than $U_{t, U} \sim U$. Therefore, neglecting U_b and solving Eq. (3) for T_b ;

$$T_b = \frac{h_{r,p-b} T_p + h_{c,b-a} T}{h_{r,p-b} + h_{c,b-a}} \quad (4)$$

Substituting Eq. (4) into Eq. (1) resulted the following equation:

$$T_a (U_L + h) = S + U_L T_a + hT \quad (5)$$

Where

$$h = h_{c,p-a} + \frac{1}{1/h_{c,b-a} + 1/h_{r,p-b}} \quad (6)$$

Substituting Eq. (4) into Eq. (2) is given by the following equation;

$$hT_p = \frac{\dot{m}}{W} c_p \frac{dT}{dx} + hT \quad (7)$$

Finally, combining Eq. (5) and (7) has resulted in the following equation:

$$\frac{\dot{m}}{W} c_p \frac{dT}{dx} = F' [S - U_L (T - T_a)] \quad (8)$$

Where is the collector efficiency factor for air collector which express in the following equation:

$$F' = \frac{1/U_L}{1/U_L + 1/h} = \frac{h}{h + U_L} \quad (9)$$

It should be mention that in all previous mathematical models which have been done by other researchers, air is assumed to behave as an ideal gas with constant specific heats. This assumption can produce some errors in thermal analysis. In a reality, the specific heat of the air varies with temperature and this will have a significant influence on thermal performance of the collector. Although one model was introduced by a team of researchers called NASA polynomial equation [18]. They have offered the following formula for the temperature range of 200-1000K:

$$\frac{C_p}{R_g} = 3.5684 - 6.7887 \times 10^{-4} T + 1.5537 \times 10^{-6} T^2 - 3.2994 \times 10^{-12} T^3 - 466.395 \times 10^{-15} T^4 \quad (10)$$

For low temperatures (275-400K) the specific-heat curve is nearly linear to a close approximation [19, 20],

$$C_p = a + bT \quad (11)$$

By use of the physical properties of air at different temperatures a and b coefficients are obtained as 980.52 and 0.083, respectively [1]. By substituting Eq. (11) into Eq. (8) and rearrangement, the following equation obtained:

$$\frac{\dot{m}}{W} a \frac{dT(x)}{dx} + \frac{\dot{m}}{W} b T(x) \frac{dT(x)}{dx} + F' U_L T(x) - F' (S + U_L T_a) = 0 \quad (12)$$

The initial condition of Eq. (12) is $T=T_i$ at $x=0$.

For calculating the convection heat transfer coefficients, Reynolds number should be determined first:

$$Re = \frac{\rho V D}{\mu} = \frac{\dot{m} D}{A_c \mu} \quad (13)$$

Where $A_c = s \times W$ is the cross sectional area and D is the hydraulic diameter of the channel:

$$D = 4 \frac{\text{Flow cross - sectional area}}{\text{Wetted perimeter}} = 4 \frac{Ws}{2W} = 2s \quad (14)$$

After determination of Reynolds number, convection heat transfer coefficients for turbulent flow can be calculated as follows [1]:

$$h_{c,p-a} = h_{c,b-a} = \frac{K}{D} 0.0158Re^{0.8} \quad (15)$$

Collector's performance can be measure using the collector thermal efficiency, defined as the ratio of useful heat gain over any time period to the incident solar radiation over the same period [21]:

$$\eta = \frac{Q_u}{A_s G_t} = \frac{\int_{T_i}^{T_o} \dot{m} c_p dT}{(W.L)G_t} = \frac{\dot{m} \int_{T_i}^{T_o} (a + bT) dT}{(W.L)G_t} \quad (16)$$

Where T_o is the outlet air temperature or temperature at $x=L$ which can be found by solving Eq. (12). As mentioned before, because of the nonlinear structure of Eq. (12), it should be solved by numerical or analytical methods which are introduced in the following sections.

Analytical Method; Differential Transformation Method (DTM): For understanding the concept of method, suppose that $x(t)$ is an analytic function in domain D and $t=t_i$ represents any point in the domain. Then the function $x(t)$ is represented by one power series which the center is located at t_i . The Taylor series expansion of $x(t)$ is shown in following equation [12]:

$$x(t) = \sum_{k=0}^{\infty} \frac{(t-t_i)^k}{k!} \left[\frac{d^k x(t)}{dt^k} \right]_{t=t_i} \quad \forall t \in D \quad (17)$$

The Maclaurin series of $x(t)$ can be obtained by taking $t_i=0$ in Eq. (17) like:

$$x(t) = \sum_{k=0}^{\infty} \frac{t^k}{k!} \left[\frac{d^k x(t)}{dt^k} \right]_{t=0} \quad \forall t \in D \quad (18)$$

As explained in literature [10] the differential transformation of the function $x(t)$ is defined as follows:

$$X(k) = \sum_{k=0}^{\infty} \frac{H^k}{k!} \left[\frac{d^k x(t)}{dt^k} \right]_{t=0} \quad (19)$$

Table 1: Some fundamental operations of the differential transform method [12]

Origin function	Transformed function
$x(t) = \alpha f(x) \pm \beta g(t)$	$X(k) = \alpha F(k) \pm \beta G(k)$
$x(t) = \frac{d^m f(t)}{dt^m}$	$X(k) = \frac{(k+m)! F(k+m)}{k!}$
$x(t) = f(t)g(t)$	$X(k) = \sum_{l=0}^k F(l)G(k-l)$
$x(t) = t^m$	$X(k) = \delta(k-m) = \begin{cases} 1, & \text{if } k = m, \\ 0, & \text{if } k \neq m. \end{cases}$
$x(t) = \exp(t)$	$X(k) = \frac{1}{k!}$
$x(t) = \sin(\omega t + \alpha)$	$X(k) = \frac{\omega^k}{k!} \sin\left(\frac{k\pi}{2} + \alpha\right)$
$x(t) = \cos(\omega t + \alpha)$	$X(k) = \frac{\omega^k}{k!} \cos\left(\frac{k\pi}{2} + \alpha\right)$

Where $X(k)$ represents the transformed function and $x(t)$ is the original function. The differential spectrum of $X(k)$ is confined within the interval, where H is a constant value. The differential inverse transform of $X(k)$ is defined as follows:

$$x(t) = \sum_{k=0}^{\infty} \left(\frac{t}{H}\right)^k X(k) \quad (20)$$

It is clear that the concept of differential transformation is based on Taylor series expansion. The values of function $X(k)$ at values of argument k are referred to as discrete, i.e. $X(0)$ is known as the zero discrete, $X(1)$ as the first discrete, etc. The more discrete available, the more precise it is possible to restore the unknown function. The function $x(t)$ consists of the T -function $X(k)$ and its value is given by the sum of the T -function with $(t/H)k$ as its coefficient. In real applications, at the right choice of constant H , the large values of argument k the discrete of spectrum reduce rapidly. The function $x(t)$ is expressed by a finite series and Eq. (20) can be written as follows:

$$x(t) = \sum_{k=0}^n \left(\frac{t}{H}\right)^k X(k) \quad (21)$$

The most important mathematical operations which are performed by differential transform method are listed in Table 1.

Applying the energy balance process, the differential equation of temperature distribution through solar collector is obtained as Eq. (12). Use of DTM, transformed form of Eq. (12) is:

$$\frac{\dot{m}}{W}a(k+1)\bar{T}(k+1) + \frac{\dot{m}}{W}b \sum_{l=0}^k (\bar{T}(k-l) - \bar{T}(k+l)) + F'U_L\bar{T}(k) - F'(S + U_L T_a)\delta(k) = 0 \quad (22)$$

Where is transformed form of T and

$$\delta(k) = \begin{cases} 1 & k=0 \\ 0 & k \neq 1 \end{cases} \quad (23)$$

Also transformed form of initial condition ($T(0)=T_i=323K$) is:

$$\bar{T}(0) = 323K \quad (24)$$

Now, if the main parameters have the following range, the DTM transformed terms of T will be obtained (by solving Eq. (22)).

$$\begin{aligned} a &= 980.52, b = 0.083, s = 0.015[m], h_{r,p-b} = 7.395[W/m^2.K], \\ \mu &= 2.051 \times 10^{-5}[kg/m.s], K = 0.029[W/m.K], \\ T_a &= 288[K], G_t = 890[W/m^2], \dot{m} = 0.06[kg/s], \\ W &= 1.2[m], U_L = 6.5[W/m^2.K], (\tau\alpha) = 0.9 \\ \bar{T}(0) &= 323, \bar{T}(1) = 8.416452738, \bar{T}(2) = -0.4043467544 \\ \bar{T}(3) &= 0.01313746939, \bar{T}(4) = -0.0003291465982, \dots \end{aligned} \quad (25)$$

By substituting DTM transformed terms of T (Eq. (25)) into Eq. (21), $T(x)$ is determined as:

$$\begin{aligned} T(x) &= 323 + 8.416452738x - 0.4043467544x^2 + 0.01313746939x^3 \\ &- 0.0003291465982x^4 + 0.6945501523 \times 10^{-5}x^5 \\ &- 1.333166033 \times 10^{-7}x^6 \end{aligned} \quad (26)$$

Numerical Method; Forth-Order Runge-Kutta Method: The numerical solution is performed using the algebra package Maple 15.0 to solve the present case. The package uses a fourth order Runge-Kutta-Fehlberg procedure for solving nonlinear boundary value (B-V) problem. The algorithm is proved to be precise and accurate in solving a wide range of mathematical and engineering problems especially heat transfer cases [22].

As it mentioned, the type of the current problem is boundary value problem (BVP) and the appropriate method needs to be selected. The available sub-methods in Maple 15.0 are a combination of the base schemes; trapezoid or midpoint method. There are two major considerations when choosing a method for a problem. The trapezoid method is generally efficient for typical problems, but the midpoint method is so capable of handling harmless end-point singularities that the trapezoid method unable to do. The midpoint method, also known as the fourth-order Runge- Kutta- Fehlberg method, improves Euler method by adding a midpoint in the step which increases the accuracy by one order. Thus, the midpoint method is used as a suitable numerical technique [23].

RESULTS AND DISCUSSIONS

To determine the temperature distribution through the air-heating solar collector, Eq. (12) should be solved. As already described, because of the nonlinearity of Eq. (12), numerical or analytical method must be applied to solve the equation. Differential transformation method and numerical method were applied to the equation. Analytical solution for special values of parameter (base values) is presented in Eq. (26). Table 2 compares the accuracy of DTM and numerical solution for the equation. It can be seen that DTM has a very good agreement with forth-order Runge-Kutta numerical method but DTM, due to its simplicity, may be introduced as better approach for solving the nonlinear differential equation like the current one.

Fig. 3 to 6 show the effect of parameters appeared in mathematical formulations on temperature distribution. Furthermore, all these figures reveal that DTM and numerical method have very close results to each other. For all these figures, following values (as the base values) are assumed and the effect of these values is investigated on each figure.

$$\begin{aligned} a &= 980.52, b = 0.083, s = 0.015[m], h_{r,p-b} = 7.395[W/m^2.K], \\ \mu &= 2.051 \times 10^{-5}[kg/m.s], K = 0.029[W/m.K], T_a = 288[K], \\ T_i &= 323[K], G_t = 890[W/m^2], \dot{m} = 0.06[kg/s], W = 1.2[m], \\ U_L &= 6.5[W/m^2.K], (\tau\alpha) = 0.9 \end{aligned}$$

Fig. 3 shows the effect of width of collector (W) on temperature. As it can be seen by increasing the width (W), temperature magnitude in the collector is increased.

Table 3: Air outlet temperature values (K) for different lengths of collector and W , $\tau\alpha$, U_L , m^{\bullet} using DTM

Parameters	Values	L(m)		
		2	4	6
$W(m)$	1.2	338.3155658	350.9593655	361.4011408
	1.7	342.7153696	358.0057679	369.8682932
	2.2	346.4830859	363.6905604	376.2958904
$(\tau\alpha)$	0.7	333.5639341	342.2874967	349.4929890
	0.8	335.9399675	346.6241070	355.4482554
	0.9	338.3155658	350.9593655	361.4011408
$U_L(W/m^2.K)$	4.5	342.1149300	358.6522082	372.9649193
	6.5	338.3155658	350.9593655	361.4011408
	8.5	335.2098480	344.8951397	352.5796182
$m(kg/s)$	0.06	338.3155658	350.9593655	361.4011408
	0.08	335.2778640	345.8379911	354.9229890
	0.1	333.2723557	342.3419626	350.3511115

Table 4: Useful gained heat ($Q_u(W)$) values for different lengths of collector and W , $\tau\alpha$, U_L , m^{\bullet} using DTM

Parameters	Values	L(m)		
		2	4	6
$W(m)$	1.2	926.2529	1691.803	2324.627
	1.7	1192.56	2118.791	2838.177
	2.2	1420.685	2463.45	3228.262
$(\tau\alpha)$	0.7	638.7593	1166.658	1602.977
	0.8	782.5052	1429.225	1963.786
	0.9	926.2529	1691.803	2324.627
$U_L(W/m^2.K)$	4.5	1156.211	2157.975	3026.083
	6.5	926.2529	1691.803	2324.627
	8.5	738.3312	1324.53	1789.963
$m(kg/s)$	0.06	926.2529	1691.803	2324.627
	0.08	989.9284	1842.161	2575.94
	0.1	1035.202	1949.925	2758.261

Effect of collector effectiveness (τ_c) is illustrated at Fig. 4. Increasing effectiveness coefficient makes the temperatures increase as a prospect and considerable result. Fig. 5 confirms that extending the heat loss coefficient (U_L) lead to decrease the temperature distribution level which is obvious. Effect of air flow rate (\dot{m}) on temperature distribution along the collector length is demonstrated by Fig. 6. A significant outcome can be concluded from this figure that when the air mass flow rate increases, because of large amount of air, temperature along the collector decreases.

For demonstrating the collector's performance, thermal efficiency must be determined through Eq. (16). So, three different length for collector are selected ($L=2, 4$ and $6m$) and the outlet temperature (T_o) from DTM formula are calculated for one of them which are shown in Table 3. As seen in Eq. (16), for calculating the collector

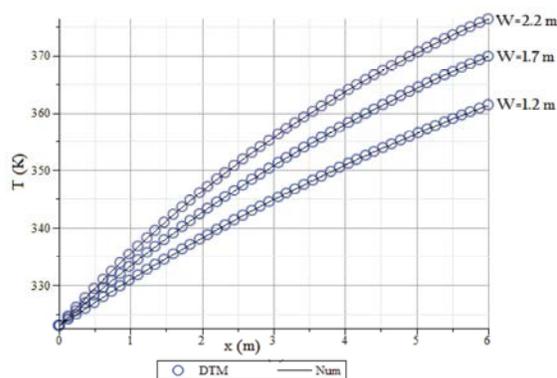


Fig. 3: Effect of W on temperature distribution along the collector by DTM and numerical method

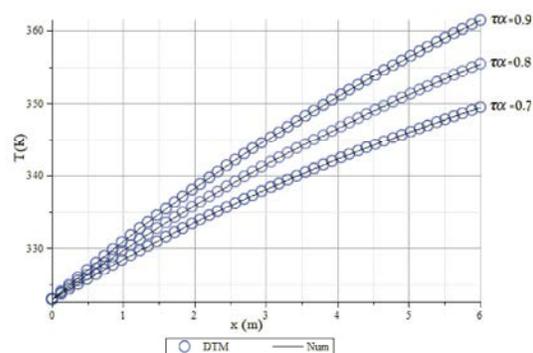


Fig. 4: Effect of $\tau\alpha$ on temperature distribution along the collector by DTM and numerical method

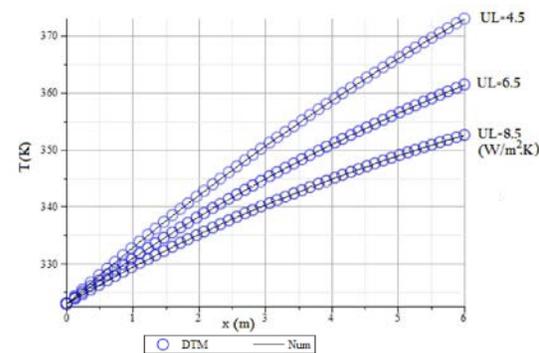


Fig. 5: Effect of U_L on temperature distribution along the collector by DTM and numerical method

efficiency the useful gained heat (Q_u) should be determined at first. Results are listed in Table 4. Using the data of Table 4, thermal efficiency of each collector can be determined.

Fig. 7 confirms that larger dimensions (L and W) for collectors have smaller thermal efficiencies. Fig. 8 demonstrates that the results of effectiveness coefficient ($\tau\alpha$) on thermal efficiency of collector for different lengths of collector.

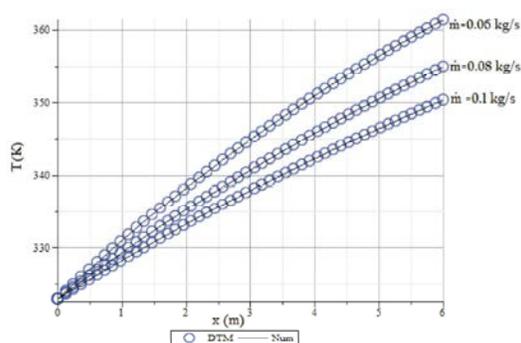


Fig. 6: Effect of air flow rate on temperature distribution along the collector by DTM and numerical

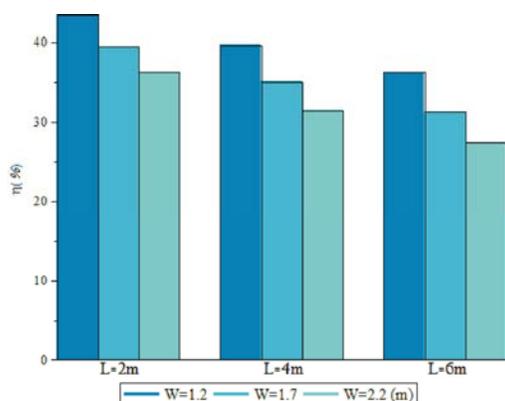


Fig. 7: Effect of length and width of the collector on its thermal efficiency

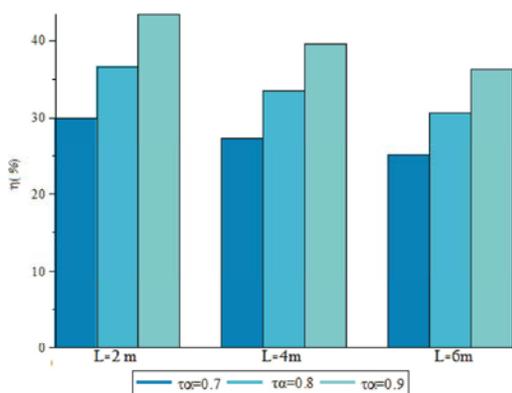


Fig. 8: Effect of length and effective ($\tau\alpha$) of the collector on its thermal efficiency

Increase in the $\tau\alpha$ leads to higher thermal efficiency but increasing in the U_L caused reduction in efficiency. Another important outcome is that although increasing in the air flow rate makes $\tau\alpha$ the temperature distribution enhanced (Fig. 6). For desired $\tau\alpha$ the thermal efficiency of the air-heating collectors increased.

CONCLUSION

Recent development on solar cells has improved the performance of solar collectors by enhanced designs. In present work, Differential Transformation Method (DTM) and numerical method were applied to obtain the temperature distribution in a flat-plate air-heating solar collector. Using analytical DTM for solving the nonlinear equation of energy through the collector and compare the results obtained by numerical approach reveals the facility, effectiveness and high accuracy of this method. Also the net gained heat and the thermal efficiency of collectors are presented with respect to the different values of main parameters which are appeared in the mathematic formula ($W, L, U_L, \tau\alpha$ and m°). A brief obtained results showed that increase in the collector size (With and Length) and the heat loss coefficient (U_L) lead to reduction in the collector efficiency. However, an increase in the effectiveness coefficient ($\tau\alpha$) and air mass flow rate (m°) were directly related to performance of solar collectors.

Nomenclature:

- a,b = Coefficients
- A_c = Cross section area of collector (m²)
- A_s = Surface area of collector (m²)
- c_p = Specific heat capacity of air (J/kg.K)
- D = Hydraulic diameter of channel (m)
- F' = Collector efficiency factor
- G_t = Total insolation (W/m²)
- H = Constant value
- $h_{c,b-a}$ = Convection heat transfer coefficient from back plate to air (W/m².K)
- $h_{c,p-a}$ = Convection heat transfer coefficient from absorber plate to air (W/m².K)
- $h_{r,p-b}$ = Radiation heat transfer coefficient from absorber plate to back plate (W/m².K)
- K = Thermal conductivity of air (W/m.K)
- k = DTM transformed form of 't'
- L = Length of collector (m)
- m° = Air mass flow rate (kg/s)
- Q_u = Useful gained heat (W)
- R_c = Reynolds number
- R_g = Gas constant (J/kg.K)
- S = Absorbed solar radiation (W/m²)
- s = Depth of air flow pass (m)
- SAH = Solar air heating
- T_i = Inlet temperature (K)
- T_a = Ambient temperature (K)
- T_b = Back plate temperature (K)

- T_o = Outlet temperature (K)
 T_p = Absorber plate temperature (K)
 \bar{T} = DTM transformed form of T
 $T(x)$ = Temperature function (K)
 t = Variable parameter in DTM
 U_b = Heat loss coefficient from back plate (W/m².K)
 U_L = Heat loss coefficient (W/m².K)
 U_t = Heat loss coefficient from absorber plate (W/m².K)
 V = Velocity (m/s)
 W = Width of collector (m)
 $X(k)$ = DTM transformed function of $x(t)$
 $x(t)$ = Analytic function
 ρ = Density (kg/m³)
 $\delta(k)$ = Dirac delta function
 δx = Element of collector length (m)
 μ = Dynamic viscosity (kg/ms)
 (τa) = Effectiveness coefficient of collector
 $(\tau \alpha)$ = Effectiveness coefficient of collector

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