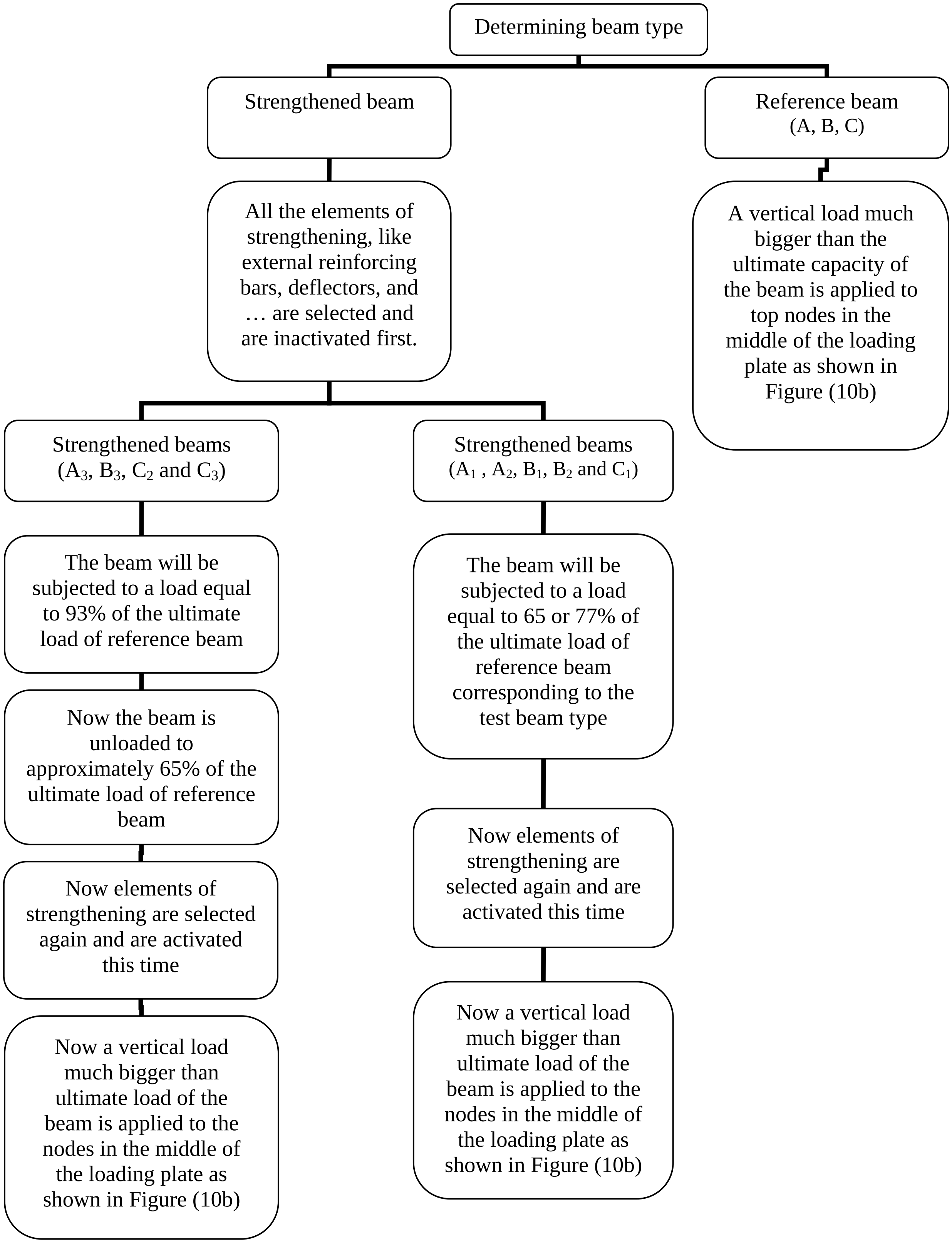

Fig. 11: Algorithm of how applying loads

Table 4: Ultimate capacity of strengthened beams prior to strengthening

Group name	Test beam name	Model name	Ultimate load *Ton*
A	A_1	*NFourteen*	10.338
	A_2	*Nfifteen*	10.91
	A_3	*Nsixteen*	9.62
B	B_1	*Neighteen*	10.066
	B_2	*Nnineteen*	8.22
	B_3	*Ntwenty*	9.216
C	C_1	*Ntwentytwo*	6.91
	C_2	*Ntwentythree*	7.35
	C_3	*Ntwentyfour*	6.517

Table 5: Results from ANSYS and tests

Test beam name	The percentage of pre-loading prior to strengthening %	The percentage of pre-loading when strengthening %	Type of beam	Ultimate load of beam *Ton* *Ansys*	*Exp*	The percentage of increase of strength *Ansys*	*Exp*	The percentage of error with respect to test
A	-	-	Reference	11.134	13	-	-	14.354
A_1	65.4	65.4	Strengthened	14.72	17	32.208	30.769	13.39
A_2	77	77	Strengthened	12.35	15.9	10.922	22.308	22.33
A_3	95	62	Strengthened	12.34	17.6	10.832	35.385	29.9
B	-	-	Reference	9.18	10.3	-	-	10.874
B_1	68	68	Strengthened	12.862	15	40.109	45.631	14.253
B_2	77.7	77.7	Strengthened	12.08	14.5	31.59	40.777	16.69
B_3	97	68	Strengthened	12.75	14	38.889	35.922	8.93
C	-	-	Reference	6.656	8.2	-	-	23.2
C_1	67	67	Strengthened	11.349	14.2	70.508	73.17	20.08
C_2	93.3	67	Strengthened	9.934	12.6	49.249	53.659	21.16
C_3	93	67	Strengthened	9.248	13.7	38.942	67.073	32.5

Table 6: Amount of load at first crack and at yielding of internal flexural tension reinforcement

Test beam name	Amount of load at first crack Ton *Ansys*	*Exp*	Amount of load at yielding of internal flexural tension reinforcement Ton *Ansys*	*Exp*
A	2.28	2	10.44	12
A_1	2	2	9.31	15~15.5
A_2	2.22	2	9.58	13
A_3	2.25	2	10.69	16.5
B	2.25	2	8	10
B_1	2.1	2	10.72	12.3
B_2	2.22	2	10.75	11
B_3	2.29	2	10.54	-
C	2.1	2	9.97	6~7
C_1	1.69	2	5.5	12
C_2	1.7	2	5.04	7~7.5
C_3	1.66	2	5.1	7.5

design load, as a major factor for study. On the other hand since ANSYS software has the capability of calculating the ultimate capacity of all beams prior to strengthening, in modeling of the strengthened beams, ultimate capacity of each beam is calculated first and then each test beam is subjected to the percentages of its own ultimate load. Table (4) shows the ultimate capacity calculated for strengthened beams prior to strengthening.

Test and Analytical Results For Beams: To verify the validity of modeling and analysis, analytical results obtained using ANSYS were compared to those obtained by test. A summery of the results is shown in Table (5).

The Amount of Load at First Crack and at Yielding of Internal Flexural Tension Reinforcement: Load-deflection diagrams can be used to determine linear region (from the beginning of the test to the attainment of the first crack of the beam), secondary linear region (from first crack to yielding of internal flexural tension reinforcement) and failure. The values of load at first crack and load at yielding of internal flexural tension reinforcement can also be determined using load-deflection diagrams. Table (6) shows the values of load at first crack and load at yielding of internal flexural tension reinforcement. The accuracy of the study can be obtained by comparison of load-deflection diagrams resulted from test and finite element

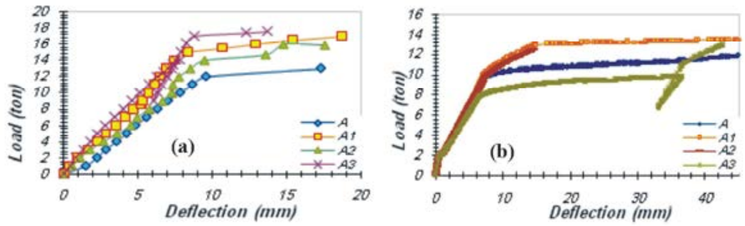

Fig. 12: Load-deflection diagram for group A beams, a) test beams, b) modeled beams

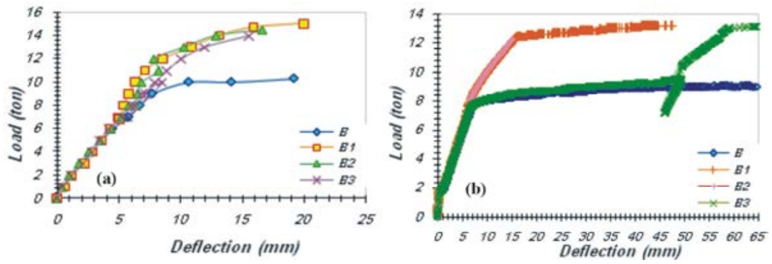

Fig. 13: Load-deflection diagram for group B beams, a) test beams, b) modeled beams

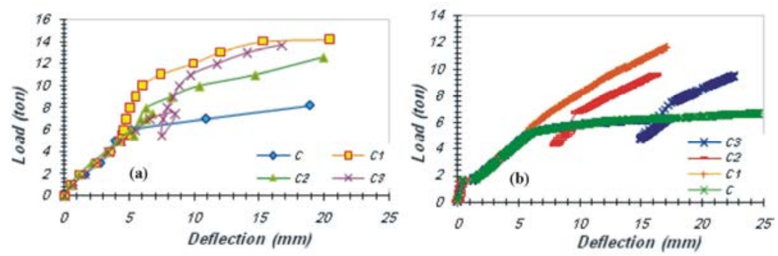

Fig. 14: Load-deflection diagram for group C, a) test beams, b) modeled beams

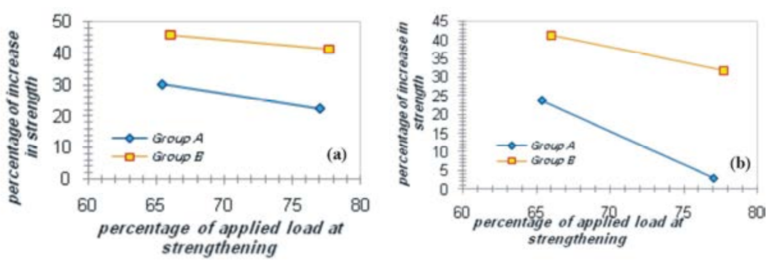

Fig. 15: Diagram of the percentage of strength increase of the beam versus the percentage of applied load at strengthening, a) test beams, b) modeled beams

analysis using ANSYS. Load- deflection diagram for group A, B and C were shown in Figures (12), (13) and (14), respectively. It can also be concluded that the rigidity of above beams were increased considerably.

Study of Strength of Specimen: Figure (15) shows the diagram of the percentage of increase in strength versus the percentage of the applied load at strengthening. The latter is a ratio of the percentage of dead load at strengthening of the beam and the ultimate load of reference beam. The diagrams show the decrease in the percentage of strength increase with respect to the increase of the applied load at the time of strengthening of the beam.

Figure (16) shows the changes in the percentage increase in strength versus the percentage of pre-loading for different percentages of internal flexural tension reinforcement in the beams. The beams were loaded to 65% of ultimate reference beam load at strengthening.

Figure (17) shows the effect of internal flexural tension reinforcement area on the percentage increase in strengthening of pre-loaded beam up to yielding of the internal flexural tension bars, while other beam parameters and the amount of external strengthening reinforcing bars are kept constant. In this diagram ρ_{ub} = 0.227 ρ_{bal} and $\frac{\rho_b}{\rho_{bal}} = K(b)$ show the ratio of internal flexural tension reinforcement and reinforcement at balance condition.

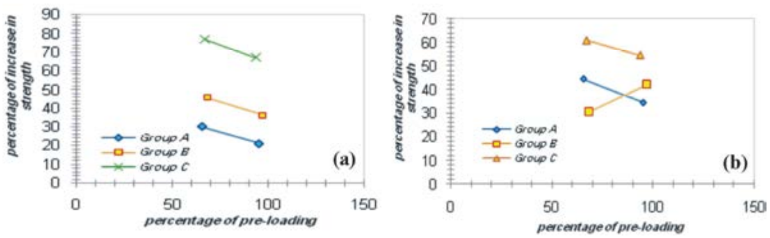

Fig. 16: Diagram of the percentage increase in strength versus the percentage of pre-loading at strengthening, a) test beams, b) modeled beams

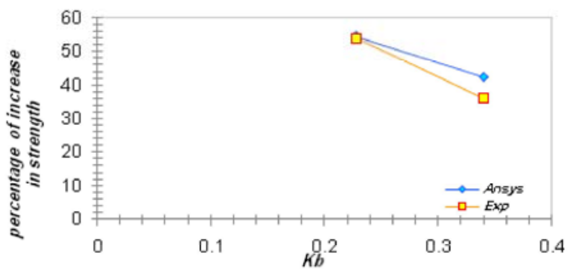

Fig. 17: Diagram of the percentage increase of strengthening versus the percentage of internal flexural tension reinforcement at $\frac{\rho_{ub}}{\rho_{bal}} = 0.227$

Study of Load-Deflection Diagrams of Specimens: Load-deflection diagrams for group A, B and C were shown in Figures (12), (13) and (14), respectively.

In load-deflection diagrams of reference beams, the beams show linear behavior up to the first crack of the beam and a nonlinear behavior after that. The deflection of the beam at the middle of the span will increase considerably with yielding of longitudinal reinforcing bars.

In strengthened beams, the load-deflection diagrams up to yielding of internal flexural tension reinforcement are similar to those of reference beams. But after yielding because of the existence of external strengthening bars, the rigidity of beams increase and depending on the amount of external strengthening reinforcing bars these external bars may yield first (if the amount of external reinforcement is low) or the concrete in compression region crushes (for large amount of external reinforcement).

Study of Specimens Rigidity: Load-deflection diagrams for group A, B and C were shown in Figures (12), (13) and (14), respectively.

Comparing load-deflection diagram of reference beam of each group with those of the rest of the beams in the same group, it can be concluded that existence of the external strengthening reinforcing bars causes a considerable decrease in deflection of the beam, i. e.; it causes an increase in beam rigidity and a decrease in beam ductility.

Study of Beams Pre-loaded over 90% of Their Ultimate Capacity: According to the results obtained in beams pre-loaded over 90% of their ultimate capacity (beams A_3, B_3, C_3 and C_2), it can be concluded that strengthening of beams by external strengthening reinforcing bars can be very effective and increases the strength of beams considerably. This holds true even in beams pre-loaded to approximately their ultimate capacity which has caused an extensive damages.

For beams pre-loaded to yielding of internal flexural tension reinforcement prior to strengthening and strengthened while sustaining 65% of ultimate load, the percentage increase in strength is decreased appreciably. This decrease is directly proportional to the amount of internal flexural tension reinforcement.

For beams pre-loaded to more than 90% of their ultimate load and then strengthened at the load approximately equal to 65% of their ultimate load, for equal amount of external strengthening reinforcing bars, the increase in strength is in reverse proportion with the amount of the internal flexural tension reinforcement.

CONCLUSION

The results of the study are summarized below:

- Strengthening of reinforced concrete beams by unbounded external reinforcing bars increases flexural strength of beams. The increase in strength is in reverse proportion with the percentage of internal flexural tension reinforcement.
- The percentage increase in strength of strengthened beams decreases with the increase in applied load at strengthening. This decrease in strength is in direction proportion with the amount of internal flexural tension reinforcement.

- The percentage increase in strength of pre-loaded beams is in reverse proportion with the amount of internal flexural tension reinforcement.
- The increase in strength increases with the increase in the percentage of external strengthening reinforcing bars.
- Strengthening of reinforced concrete beams by external strengthening reinforcing bars can be used as an effective method even in cases where beams are under load. The advantages of the method are; speedy application, simplicity of employment, almost no increase in weight of the structure and economic advantages.
- Since there exists close agreement between load-deflection diagrams of modeled and test beams, the models constructed can be used for future research and the model is valid for modeling of reinforced concrete beams strengthened by external strengthening bars.
- The results show that in most beams, there is a close agreement between the load-deflection diagrams of modeled and test beams, especially before the yielding of internal flexural tension reinforcement.

Notations:

a_v	Shear span	mm
A_{sb}	Cross sectional area of bonded reinforcement mm^2	
A_{sub}	Cross sectional area of unbounded reinforcement	mm^2
b	Width of beam	mm
d	Effective depth	mm
d'	Compression effective depth	mm
d_{ub}	Unbounded effective depth at ultimate load	mm
E_c	Elastic modulus of concrete	N/mm^2
E_s	Elastic modulus of steel	N/mm^2
f_c	Cylindrical compressive strength of concrete	N/mm^2
f_{cu}	Compressive strengths of concrete	N/mm^2
f_{sb}	Stress in bonded reinforcement	N/mm^2
f_{st}	Post tensioned stress	N/mm^2
f_{sub}	Stress in unbounded reinforcement	N/mm^2
f_{yb}	Yield strength of bonded tension reinforcement	N/mm^2
f_{yc}	Yield strength of compression reinforcement	N/mm^2
I	Moment of inertia of cracked section	mm^4
K	Defined factor	
K_1	Defined factor	
K_2	Defined factor	
K_3	Defined factor	
L	Length of beam	mm
L_1	Beam span	mm
M	Flexural moment in specified section	$N.mm$
M_u	Ultimate moment	$N.mm$
M'_u	Defined factor	
P_u	Ultimate load	N
Q_b	Defined factor	
Q_c	Defined factor	

Q_t	Defined factor	
Q_{ub}	Defined factor	
x	The distance of the extreme compressive fiber of concrete from neutral axis of the beam	*mm*
z	The internal lever arm	*mm*
β	Defined factor	
Δ	Deflection at service load	*mm*
ε_c	Strain in concrete at the level of reinforcement	
ε_{sb}	Strain in bounded reinforcement	
ε_{sub}	Strain in unbounded reinforcement	
ρ_b	Bonded tension reinforcement ratio	
ρ_c	Compression reinforcement ratio	
ρ_{ub}	unbounded reinforcement ratio	

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