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Analytical Modeling for Prediction of Horizontal-Axis Wind Turbines Power Generation in Wind Farms Based on an Analytical Wake Model

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A B S T R A C T

The development of models that predict power production of wind farms (WFs) by considering the interacting wakes is important; because wakes of the turbines exert a significant influence on power production of turbines, and hence on the layout of wind turbines in WFs. Thus, the purpose of present study was to provide an innovative analytical method for the prediction of power generation of the WFs that have a flat terrain and are consisted of horizontal-axis wind turbines (HAWTs) with the same hub height. The methodology employed utilized an analytical Gaussian model of HAWT wake to develop an analytical model that calculates the effective wind velocity acting on the downstream HAWT(s), which is further used for reading its generated power from the turbine's catalog; thus, providing the generated power of the WF as the output. The results of presented model were validated by the field measurements data of Horns Rev WF and also were compared to two analytical models for predicting the generated power. The results were compared with two numerical simulations of the literature, and the output data of three commercial software. Moreover, the error analysis revealed that the presented model mostly showed superior accuracy in predicting the field measurements data.

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Nomenclature

C_P	Efficiency coefficient of turbine	$u_{\infty,z}$	Wind 2d vertical velocity (free stream) profile (m s^{-1})
C_T	Thrust coefficient turbine	u_{eff}	Upstream turbine i wake velocity profile effective on the downstream wind turbine j (m s^{-1})
d_t	Diameter of turbine (m)	u_{ref}	Wind velocity (free stream) measured at z_{ref} (m s^{-1})
$d_{w,0}$	Initial diameter of wake (m)	$u_{w,i,z,y,x}$	Wake 3d velocity profile of upstream turbine i at x (m s^{-1})
$d_{w,x}$	Diameter of wake at x (m)	$u_{w,r,x}$	Wake 2d velocity profile at x (m s^{-1})
I_∞	Turbulence intensity of the incoming wind	$u_{w,\bar{r},x}$	Mean velocity of wake at x (m s^{-1})
I_{wake}	Turbulence intensity of wake	$u_{w,z,y,x}$	Wake 3d velocity profile (m s^{-1})
k_w	Decay coefficient of wake	x	Downstream length from turbine (m)
$k_{w,rev}$	Revised coefficient of wake decay	y	Spanwise length from turbine (m)
r	Length from center of wake (m)	z	Vertical length from the ground (m)
r_t	Radius of turbine (m)	z_h	Hub axis length from the ground (m)

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$r_{t,j}$	Radius of downstream wind turbine j (m)	z_{ref}	Reference height for measuring u_{ref} (m)
$r_{w,0}$	Initial radius of wake (m)	**Greek symbols**	
$r_{w,x}$	Radius of wake at downstream length x (m)	α	Wind shear index
u	Velocity profile (3d) of the wind in the field (m s^{-1})	θ_{wind}	Direction of the free stream wind (°)
u_{∞}	Wind velocity (free stream) at hub (m s^{-1})		

INTRODUCTION

Replacing fossil-based production of electricity with the generation of electricity through the use of renewable sources of energy has become one of the energy priorities of the governments, due to ozone depletion, climate change, acid rain, CO_2 emissions and rising electricity prices [1, 2]. According to the statistics of the International Energy Association (IEA) [3], from 1990 to 2015, the average every-five-year growth in the electricity generated by renewable (non-combustible) energy sources surpassed that of fossil-based electricity generation (18.5% vs. 16.6%, respectively). In 2018, renewable energy (non-combustible) constituted 14.3% of the total generation of electricity. Out of this, 2.9% was the share of wind energy, which puts into spotlight the importance of wind energy as one of the main sources of renewable energy, and emphasizes the role of wind turbines as the means of wind power harvesting.

In wind turbines, electricity is generated from the transformation of wind energy. Extraction of the wind energy and the rotor drag induce cuts in the wind's speed and energy in a region downstream of the turbine, called the turbine wake [4]. As a result, the turbines implemented in the wake of the upstream turbines generate less power [5], and their mechanical structures might be exposed to a heightened level of fatigue [6]. Thus, in order to obtain the most power output from the wind farms, it is essential to predict the power generation of the implemented wind turbines by taking into account the generated wake of the upstream turbine(s).

Methods for the prediction of the wind velocity distribution in the wake are classified into three categories, namely analytical, experimental and numerical modeling [7]. Experimental modeling suffers from restrictions in the analysis of some of the parameters such as velocity and pressure [8] and is not considered efficient enough in the optimizations of wind turbines arrangements in the wind farms [9]. The computational costs of the numerical models are rather high, but they provide relatively precise results for the wake velocity distribution [10]. Analytical modeling can result in adequate levels of precision by adopting simple equations at ~10^6-10^7 less CPU time per run compared to numerical models [7], such as LES [7], which makes employment of this kind of modeling suitable in commercial software [9].

Some of the studies examining wake by conducting analytical modeling as reported in literature [8, 11-13] (a brief description of these studies is discussed later in section literature review). It is of note that the most accurate analytical models for predicting the wake velocity are made upon the bell-shaped wind velocity distribution, and mainly rely on Gaussian distribution functions to provide an accurate wind velocity distribution of the wake (e.g., in research conducted by Ge et al. [14], Bastankhah and Porté-Agel [15], Xiaoxia et al. [16], Ishihara and Qian [17]). The Gaussian velocity distribution uses the Gaussian function as $f\exp(-0.5(r/c)^2)$ to derive the velocity deficit inside the wake and analytically model the wake velocity profile (Figure 1) [13]. In this type of model, the value of the wake velocity profile at the downstream distance x is minimum at the wake center (which is aligned with the hub axis) and increases to its maximum value at the boundary of the wake by following the Gaussian function.

Recently the subject of predicting the power generation of wind turbines has become the focus of the wind turbines research, where most of such studies have been carried out by employing functional analysis e.g. discrete wavelet transform discussed in literature [18]; signal decomposition techniques [19] or artificial intelligence [20] that were not primarily based on principles of mechanical engineering. Thus, objective of the present paper was to develop an analytical model for the prediction of the power generation of flat-terrain wind farms that contain horizontal-axis wind turbines (HAWTs) with the same hub height that are placed in each other’s wake. This was achieved by using a novel

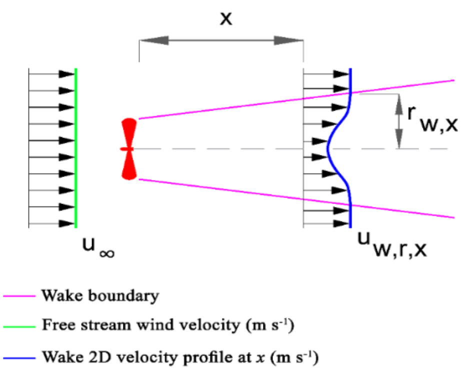

Figure 1. Wind velocity distribution in the wake [13]

3D Gaussian analytical model of HAWT wakes, which was presented in our previous work [13], and is utilized for calculating the effective wind velocity acting on the downstream HAWT, which was carried out by a novel method. The latter parameter is further used for reading the generated power of HAWT from the power curve, which is available from the manufacturer. Thus, the output of the analytical model would be the generated power of the wind farm. In the next step, the obtained power-predicting model was validated using the field measurements data of Horns Rev wind farm at two different angles of the incoming free-stream wind (270° and 222°) [21]. Finally, the results obtained from the presented model were compared to the results of other Gaussian-based power-predicting analytical models [22, 23], the numerical data (derived from standard k-ε modeling [24] and LES modeling [25]) and the available output data of commercial software which was obtained from WAsP and WindSim simulations [21], and GH WindFarmer simulation [25].

LITERTAURE REVIEW

Examples of studies used analytical models for investigating the wake were reported in literature [8, 11-13]. Cheng et al. [8] used Obukhov length, Gaussian function and surface roughness length to calculate the wake expansion and velocity deficit, in order to derive the wind velocity profile in the wake. Tian et al. [11] proposed a model that calculates the width of the wake and the wake turbulence level at any distance downstream of turbines. Also, the anisotropic characteristics of the wake were included in the model by introducing the impacts of the ground and wind shear component into the proposed model. Li et al. [12] estimated the streamwise turbulence intensity of the wake through the self-similar characteristic of the added turbulence intensity and its linear expansion rate downstream of the turbine. Furthermore, to obtain more accurate results, a novel function was also proposed to include the effects of the ground on the wake. Ayoubi et al. [13] proposed a simple 3D velocity model for the wind velocity distribution inside the wakes. The presented model was based on the Gaussian function and an isotropic value for the wake expansion rate.

For the prediction of the generated power of the wind turbines implemented in wind farms, Wang et al. [22], Niayifar and Porté-Agel [23] presented analytical models that were based on the Gaussian velocity distribution of the wake. The power-predicting analytical model of Wang et al. [22] was primarily based on a Gaussian wake model that was previously presented by Bastankhah and Porté-Agel [15]. It is of note that this model of Bastankhah and Porté-Agel has been found to give less accurate results in predicting the wake velocity of wind turbines in comparison to other Gaussian models [13]. Also, one should point out that the model of Wang et al. [22] utilizes the overlapping ratio of the area between the downstream turbine(s) and the wake of the upstream turbine(s) for deriving the velocity deficit acting on the downstream wind turbine(s). For deriving the velocity deficit acting on the downstream turbine, in addition to taking into account the overlapping ratio of the area between the downstream turbine(s) and the wake of the upstream turbine(s), they also assumed a linear rate for the wake growth was represented by Jensen [26].

In the model presented by Niayifar and Porté-Agel [23], they assumed operating conditions, in which, the thrust coefficient of wind turbines (C_T) was approximately constant. This assumption makes their model practical only within a limited range of the wind velocity, where a constant value for C_T could be assumed. Moreover, in their prediction of the wind velocity acting on the downstream wind turbines (by taking into account the wakes generated by upstream wind turbines), Niayifar and Porté-Agel [23] used the linear superposition method, which was later found to be an inaccurate method through the results obtained by Tian et al. [21]. Also, in their presented model, the turbulence intensity of the incoming wind (I_∞) should be within the range of $0.065 < I_\infty < 0.15$. Moreover, their model is only applicable, where the turbines spacing (x) is within the range $5d_t < x < 15d_t$. All of these impose restrictions on the applicability of their model.

METHODOLOGY

Velocity profile for a single wake

To derive the analytical power-predicting model, the 3D wake velocity profile ($u_{w,z,y,x}$) presented in the previous research, Equation (1), was used [13] (for the definitions of the symbols, we refer to the nomenclature). The first term on the right-hand side of Equation (1) is the 2D free stream wind velocity profile in the vertical direction, which was calculated by Equation (2) [27]. The second term on the right-hand side of Equation (1) is the Gaussian velocity deficit profile that was calculated by Equations (3) to (9). Equation (3) was derived by applying the boundary condition at $z = z_h$, where we have $u_{w,z,y,x} = 0.99u_{\infty,z}$ [13]. Equations (4) to (7) were derived by considering the conservation of mass and momentum, and the Gaussian distribution velocity profile of the wake within the control volume as shown in Figure 2 (Equations (4) and Equation (7) [16], and Equations (5) and (6) [13]). Equation (8) calculates the turbulence intensity of the wake at $z = z_h$ (I_{wake}) [13], and Equation (9) calculates the revised value of the wake decay coefficient ($k_{w,rev}$) [28]. For more details on the procedures of deriving these equations, we refer to our previous work [13].

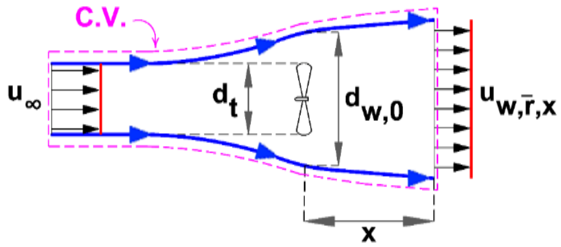

Figure 2. The chosen control volume. The blue and purple lines represent the streamlines and the boundary of the control volume, respectively [13].

$$u_{w,z,y,x} = u_{\infty,z} - fe^{-\frac{1}{2}(\frac{(z_h-z)^2+y^2}{c^2})} \quad (1)$$

$$u_{\infty,z} = u_{ref}(\frac{z}{z_{ref}})^{\alpha} \quad (2)$$

$$f = 0.01u_\infty e^{\frac{1}{2}(\frac{r_{w,x}}{c})^2} \quad (3)$$

$$r_{w,x} = k_{w,rev}x + r_{w,0} \quad (4)$$

$$r_{w,0} = r_t(\frac{u_\infty}{u_{w,0}})^{0.5} = r_t(1-C_P)^{-0.25} \quad (5)$$

$$\int_{-r_{w,x}}^{+r_{w,x}} u_{w,r,x}dr = \int_{-r_{w,x}}^{+r_{w,x}} (u_\infty - fe^{-\frac{1}{2}(\frac{r}{c})^2})dr = 2u_{w,\bar{r},x}r_{w,x} \quad (6)$$

$$u_{w,\bar{r},x} = u_\infty(1-\frac{1-\sqrt{1-C_T}}{(1+k_{w,rev}x/r_{w,x})^2}) \quad (7)$$

$$I_{wake} = ((k_n\frac{C_T}{x/d_t})^{3/4} + I_\infty^{3/4})^{4/3} \; ; \; k_n = 0.4 \quad (8)$$

$$k_{w,rev} = k_w\frac{I_{wake}}{I_\infty} \quad (9)$$

Since the purpose of this paper was to present a model for the prediction of the power generation of wind turbines with equal hub heights, the distance from the ground would equal the turbines hub height ($z = z_h$). Thus, the simplified 2D form of Equation (1) can be used, which is defined as Equation (10):

$$u_{w,r,x} = u_\infty - fe^{-\frac{1}{2}(\frac{y}{c})^2} \quad (10)$$

Multiple-interacting wake velocity field

For the calculation of the field wind velocity profile u at turbines hub height for the cases of interacting wakes, any of the following four analytical methods could be utilized [29]:

Geometric sum (GS): $$\frac{u}{u_\infty} = \prod_{i=1}^{n}\frac{u_{w,i,r,x}}{u_\infty} \quad (11)$$

Linear superposition (LS): $$1-\frac{u}{u_\infty} = \sum_{i=1}^{n}(1-\frac{u_{w,i,r,x}}{u_\infty}) \quad (12)$$

Energy of balance (EOB): $$u_\infty^2 - u^2 = \sum_{i=1}^{n}(u_\infty^2 - u_{w,i,r,x}^2) \quad (13)$$

Root sum of squares (RSS): $$(1-\frac{u}{u_\infty})^2 = \sum_{i=1}^{n}(1-\frac{u_{w,i,r,x}}{u_\infty})^2 \quad (14)$$

where $u_{w,i,r,x}$ is the wake velocity profile of the upstream turbine i at its hub. Employment of any of these methods requires the calculation of the wake velocity profile for each effective upstream turbine ($u_{w,i,r,x}$). According to Tian et al. [21], the best agreement with measured field data of multiple-implemented turbines was obtained for the RSS method. Thus, we utilized the RSS method [Equation (14)] for obtaining the field wind velocity profile u for the case of multiple-wake interaction.

Prediction of wind farm power generation

We have adopted an innovative analytical method for predicting the power produced by a wind turbine. This method is based on using the power curves of turbines, which are available from the turbine manufacturer. These power curves provide power as a function of the free stream wind velocity. In wind farms, a turbine placed on the second row or beyond might be affected by the wake(s) of the upstream wind turbine(s) (Figure 3). For such a wind turbine (downwind wind turbine j), instead of utilizing the free stream wind velocity at hub height (u_∞) or the field wind velocity profile (u) at the wind turbine position for obtaining its generated power from the power curve, we proposed utilizing the effective wind velocity acting on the downstream wind turbine j (u_{eff}) obtained by the RSS method [Equation (15)]. Thus, u_{eff} is defined as Equation (16):

$$u = u_\infty(1-\sqrt{\sum_{i=1}^{n}(1-\frac{u_{w,i,r,x}}{u_\infty})^2}) \quad (15)$$

$$u_{eff} = \frac{\int_{-r_{t,j}}^{+r_{t,j}} udy}{2r_{t,j}} \quad (16)$$

In other words, u_{eff} is the average of the wind velocities acting on the downstream turbine j at $z = z_h$. In the present work, this parameter was used as the wind velocity in extracting the power output from the power curves of the turbines. Moreover, a MATLAB code was developed for computing the generated power of the turbines of a wind farm. The code's inputs are the positions of the wind turbines, their characteristics and power curves, and the properties of the field. Then, the code performs the calculations for (i) the wake velocity profile of upstream turbine(s) at the position(s) of the downstream turbine(s) (Equation (10)), (ii) the field wind velocity profile at the position of each turbine by taking into account the interacting wakes (Equation (15)), (iii) the effective wind velocity acting on each turbine (Equation (16)), and (iv) the generated power of each turbine by using its power curve. Thus, the output of the code is the generated power of each turbine. The process of the calculations is shown in Figure 4.

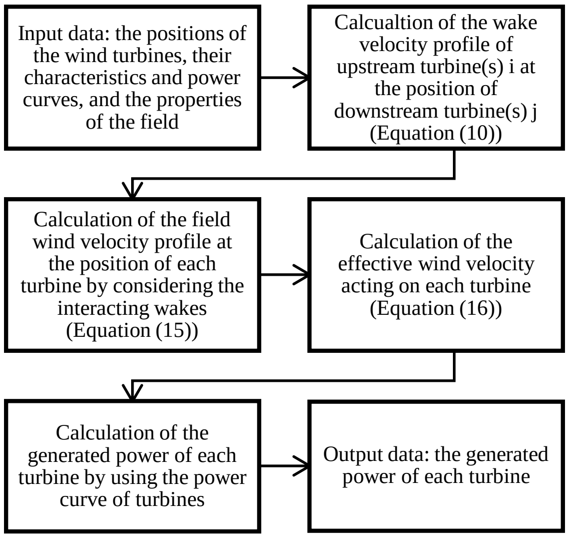

Figure 4. The process of the calculations used by the developed code

ANALYSIS

Characteristics of Horns Rev wind farm

For validating the ability of the presented model in predicting the power generation of wind farms, the field measurements data of Horns Rev wind farm have been used [21]. The location of the Horns Rev wind farm is the North Sea 14 km off the west coast of Denmark. This wind farm is made up of 80 wind turbines manufactured by Vestas (V80-2 MW) arranged in a matrix shape of 8 columns (south to north) by 10 rows (east to west) (Figure 5). In this wind farm, the inter-turbine distance in both the columns and the rows is $7d_t$ [21]. The Vestas-V80 wind turbine implemented in the Horns Rev wind farm is a 2 MW, pitch-controlled, variable-speed wind turbine with a rotor diameter of 80 m and a hub height of 70 m. In Figure 6, the manufacturer's curves for the power and the thrust

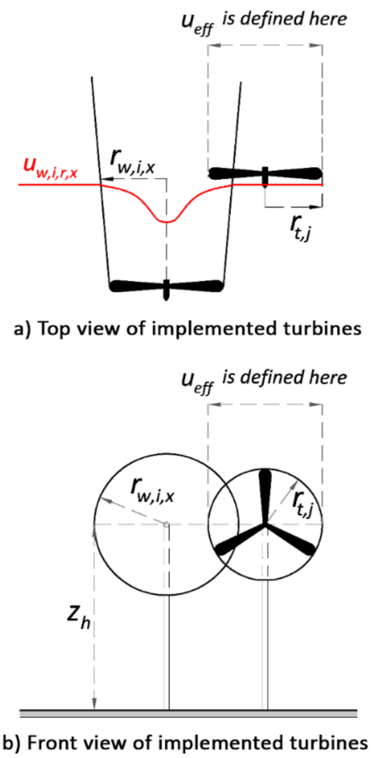

Figure 3. The area within which u_{eff} is defined, a) the top view and b) the front view of implemented turbines

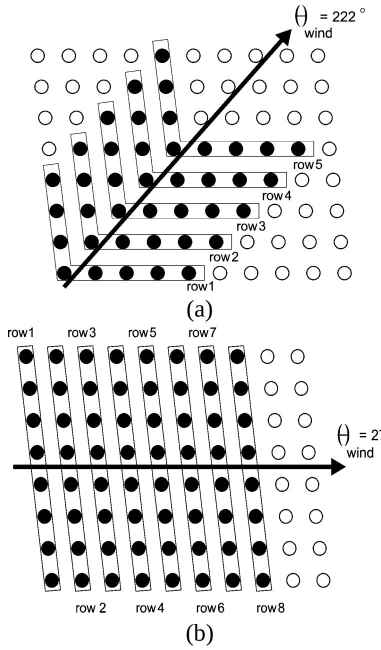

Figure 5. Turbines layouts in Horns Rev and those considered for (a) Case 1, $\theta_{wind} = 222^{\circ}$ and (b) case 2, $\theta_{wind} = 270^{\circ}$ [21]

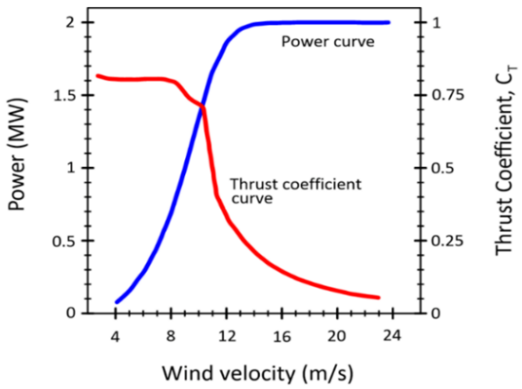

Figure 6. Thrust coefficient and power curves of the considered turbine (Vestas V80-2 MW) [21]

coefficient as functions of the wind speed are presented. A low level of turbulence (<8%) and many operating hours in a nearly neutral stability state are characteristics of this wind farm. The data set of Horns Rev used here is consisted of representative 10-min measured values from each wind turbine, and the wind speed utilized for our validation is derived from the power measurements and the V80 power curve [21]. Characteristics of the associated turbine and field are summarized in Table 1 (for more details refer to literature [21]).

Description of the results: Cases 1 and 2

The prediction of the turbines' power generation was carried out for two different angles of 222° (Case 1) and 270° (Case 2) for the incoming free-stream wind (Figure 5). The obtained results of the presented power-predicting analytical model were compared to the field measurements data [21], the available output data of commercial software, namely WAsP and WindSim [21], and GH WindFarmer [25] for Case 2 (Figure 7), the results of the power-predicting analytical models of Wang et al. [22], Niayifar and Porté-Agel [23] (Figure 8), and the numerical data of the LES study of Wu and Porté-Agel [25] and the standard k-ε study of Naderi et al. [24] (Figure 9) for Cases 1 and 2. It should be noted that in the presented data (Figure 7 to Figure **9**), the normalized power is the ratio of the average generated power of a turbine row to the average generated power of the first turbine row. Since the value of the normalized power of the first turbine row is equal to 1, we have omitted the data of the first turbine row from the plots for a better depiction of the normalized power of the downstream rows (Figure 7 toFigure 9).

Table 1. Characteristics of the turbine and field used for validation of power estimation of turbines of Cases 1 and 2

Case	θ_{wind} (°)	u_∞ (ms^{-1})	I_∞	d_t (m)	z_h (m)	k_w	C_T
Case 1	222	8.00	0.077	80	70	0.069	0.8
Case 2	270	8.00	0.077	80	70	0.069	0.8

The output results of the commercial software (available for Case 2) are shown in Figure 7. As can be seen in Figure 7, the results of WAsP and WindSim show a steady flat trend for row number 4 and beyond, whereas the results of GH WindFarmer show a decreasing trend in the prediction of the generated power of the turbines. In both Cases 1 and 2, the results of the presented model show a slight change in the power generation as the row number increases (Figure 8 to Figure **9**). In contrast, the results obtained from the analytical model of Wang et al. [22] show a steady decreasing trend as the row number increases (Figure 8). Conversely, in the analytical model of Niayifar and Porté-Agel [23], there is an increase in the predicted power generation once the row of the wind turbines increased from 2 to 3. However, there is a decreasing trend for the predicted power generation beyond row number 3 (Figure 8). In evaluating the data obtained from the numerical studies, the standard k-ε data show a steady decreasing trend in both Cases 1 and 2 (Figure 9), whereas the LES modeling almost results in a flat, steady trend in power prediction, except for row number 2 in Case 2 (Figure 9b).

Analysis of the results: Cases 1 and 2

For comparing the effectiveness of our model, the two analytical models of the literature, the results of the numerical studies and the output data of the commercial software in predicting the field measured data of the power generation, we used the value of the mean absolute percentage error (MAPE), which is defined as follows:

$$\mathrm{MAPE} = \frac{100}{n}\sum_{t=1}^{n}\left|\frac{A_t - F_t}{A_t}\right| \quad (17)$$

where A_t, F_t and n are the actual value (i.e., the field measurements data), the predicted values (i.e., the values

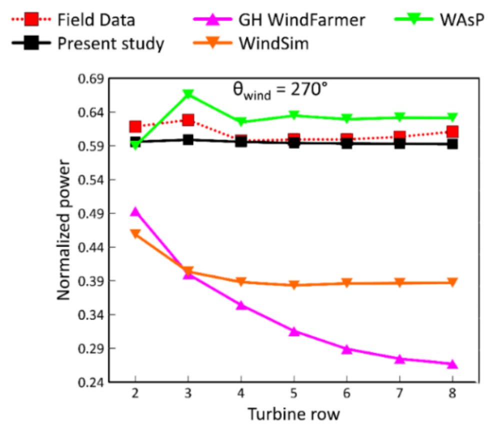

Figure 7. The results obtained from the presented model and commercial software for the power generation of Horns Rev wind farm for Case 2, $\theta_{wind} = 270^\circ$

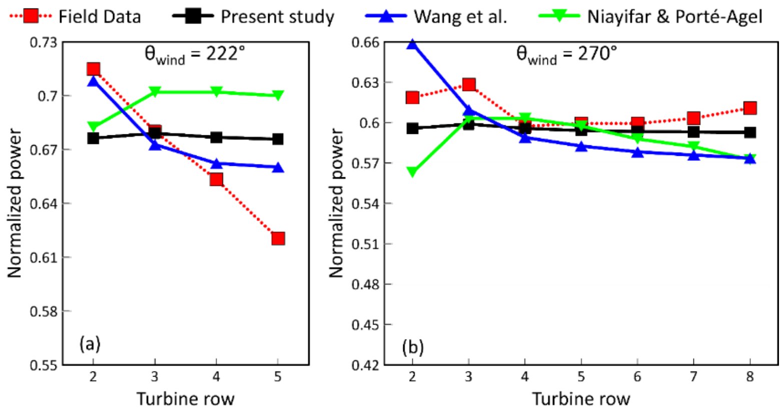

Figure 8. The results obtained from the presented model and other analytical models for the power generation of Horns Rev wind farm for (a) Case 1, $\theta_{\text{wind}} = 222^{\circ}$ and (b) case 2, $\theta_{\text{wind}} = 270^{\circ}$

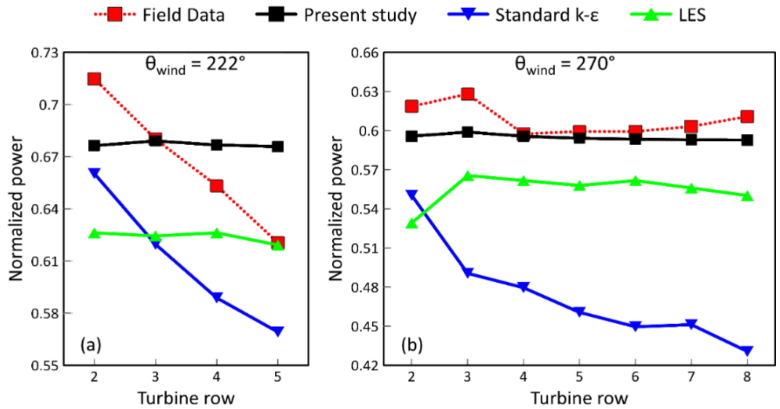

Figure 9. The results obtained from the presented model and numerical models for the power generation of Horns Rev wind farm for (a) Case 1, $\theta_{\text{wind}} = 222^{\circ}$ and (b) case 2, $\theta_{\text{wind}} = 270^{\circ}$

obtained by the modeling), and the number of the fitted points, respectively.

The results of the MAPE analysis of the normalized power for Cases 1 and 2 are presented in Figure 10 to Figure 12. By condering the results of the commercial software, it should be added that among the avg. MAPE values obtained for the output data of the commercial software used for Case 2 (Figure 10), only that of WAsP exhibited a good level of accuracy (avg. MAPE = 4.24%) and the rest indicated very large deviations from the field measurements data. By comparing the extent of the accuracy of the analytical models considered in predicting the field measured data of the power generation, as can be seen in Figure 11, for Case 1 our model came second after Wang et al. [22] model [avg. MAPE = 3.61 and 2.61%, respectively] and for Case 2 its prediction (i.e., avg. MAPE = 1.89%) was superior to those of the other models (e.g., avg. MAPE = 3.24% for Wang et al. [22] model). It is of note that compared to the numerical models and the commercial software considered, our model gave the lowest value for avg. MAPE for both Case 1 (Figure 12a) and Case 2 (Figure 12b and Figure 10).

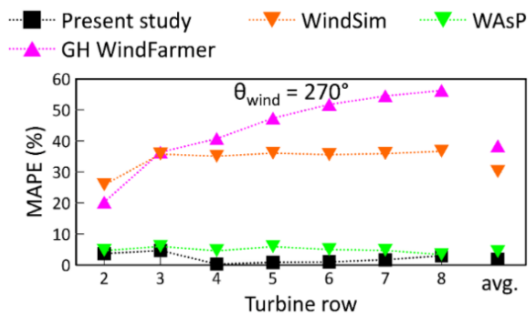

Figure 10. Values of MAPE at diferent turbine rows and the average (avg.) MAPE of rows number 2 and beyond for the results obtained from the presented model and commercial software for the power generation of Horns Rev wind farm for Case 2, $\theta_{\text{wind}} = 270^{\circ}$

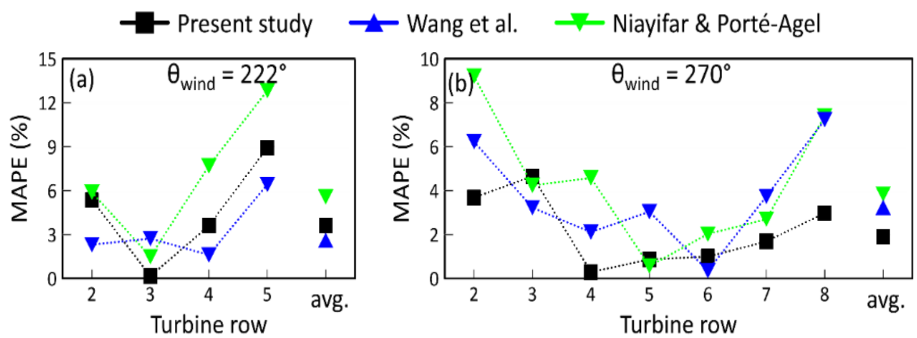

Figure 11. Values of MAPE at diferent turbine rows and the average (avg.) MAPE of rows number 2 and beyond for the results obtained from the presented model and other analytical models for the power generation of Horns Rev wind farm (a) Case 1, $\theta_{\text{wind}} = 222^{\circ}$ and (b) case 2, $\theta_{\text{wind}} = 270^{\circ}$

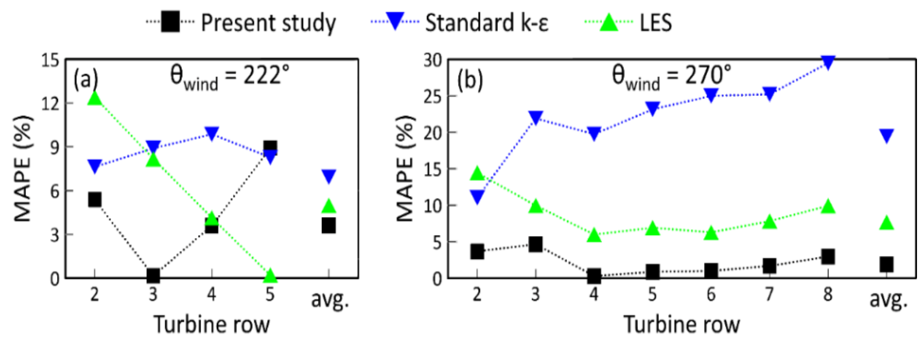

Figure 12. Values of MAPE at diferent turbine rows and the average (avg.) MAPE of rows number 2 and beyond for the results obtained from the presented model and numerical models for the power generation of Horns Rev wind farm for (a) Case 1, $\theta_{\text{wind}} = 222^{\circ}$ and (b) case 2, $\theta_{\text{wind}} = 270^{\circ}$

CONCLUSION

This research presented an analytical model for predicting HAWT power generation that was based upon the Gaussian description of the 3D velocity profile of the wake. By adopting a novel method, the 3D velocity profile of the wake was utilized for calculating the effective wind velocity acting on the downstream HAWT for the case of multiple-interacting wakes in wind farms. The obtained value for the effective wind velocity was further used for reading the generated power of HAWT from the power curve, which is available from the manufacturer of the turbine. Moreover, a MATLAB code was developed based on the presented model for computing the generated power of the turbines of a wind farm, where the positions of the wind turbines, their characteristics and power curves, and the properties of the field were taken as the initial inputs.

The ability of the presented power-predicting model in the prediction of wind farm power generation was validated through field measurements data of 64 wind turbines (each having a capacity of 2 MW) implemented in Horns Rev wind farm for two different angles of 222° (Case 1) and 270° (Case 2) of the incoming free-stream wind. Moreover, the results obtained from the presented model have been compared to the results from the two analytical models of the literature for Cases 1 and 2, the numerical data of two previous studies (LES data and standard k-ε data) for Cases 1 and 2, and output data of three commercial software (GH WindFarmer, WinSim and WAsP) of previous investigations for Case 2. Evaluation of the values of the mean absolute percentage error (MAPE) revealed that the presented model mostly showed a superior accuracy (avg. MAPE = 3.61 for Case 1, and avg. MAPE = 1.89% for Case 2) in predicting the power generation compared to other analytical power-predicting models, numerical studies and commercial software.

Since the presented model is easy to implement and also renders rather high accuracy results, this model can be used in studies concerning the implementation of wind turbines such as predicting the power production of wind farms in the initial feasibility studies, implementation of new wind turbines among the previously implemented turbines of a wind farm, and also the layout optimization of wind turbines in wind farms.

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CONFLICT OF INTEREST

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Persian Abstract

چکیده

توسعه مدل‌های مزارع بادی که با در نظر گرفتن برهمکنش ویک‌های توربین‌های بادی قابلیت پیش‌بینی توان تولیدی مزرعه را داشته باشند، امری مهم به حساب می‌آید چرا که ویک‌های توربین‌های بادی تاثیر بسزایی بر توان تولیدی توربین‌های بادی دارد. هدف مطالعه حاضر ارائه یک روش تحلیلی ابتکاری جهت پیش‌بینی توان تولیدی مزارع بادی مسطح می‌باشد که مجهز به توربین‌های بادی محور افقی با ارتفاع هاب یکسان هستند. روش به کار گرفته شده بر مبنای یک مدل گوسی ویک توربین‌های بادی محور افقی می‌باشد که پیش از این ارائه شده بوده است. بر مبنای این مدل ویک، یک مدل تحلیلی به دست آمده است که سرعت باد موثر بر توربین(های) بادی محور افقی پایین دستی را محاسبه می‌کند که در قدم بعد از آن برای استخراج توان تولیدی توربین(ها) از منحنی قدرت (که توسط سازنده توربین فراهم می‌شود) مورد استفاده قرار می‌گیرد. به این ترتیب خروجی مدل تحلیلی توان تولیدی مزرعه بادی خواهد بود. اعتبارسنجی مدل ارائه شده با استفاده از داده اندازه‌گیری شده در مزرعه بادی هورنز ریو برای دو زاویه جریان باد ورودی انجام گرفته است. تحلیل خطای نتایج نشان داد که دقت مدل ارائه شده در پیش‌بینی داده اندازه‌گیری مورد بحث بیش از دو مدل تحلیلی پیشین و نتایج دو مطالعه شبیه‌سازی و همچنین داده خروجی سه نرم‌افزار تجاری می‌باشد.